Quarterly Journal of Finance
Vol. 6, No. 1 (2016) 1650002 (59 pages)

DOI: 10.1142/S2010139216500026

Idiosyncratic Volatility of Small Public Firms and Entrepreneurial Risk

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Accepted 15 April 2015
Published 29 July 2015

The average idiosyncratic volatility of small public firms is a positive predictor of future stock returns. This is true for returns of both large and small firms. We consider several economic arguments for this result, including a liquidity premium, and we rule out all but one of them. Our evidence supports the *entrepreneurial risk hypothesis*, which states that small firms' idiosyncratic risk is a proxy for risk faced by private business owners, who also happen to be significant shareholders of stock. Expected returns are increasing functions of entrepreneurial risk, and therefore returns are predictable using proxies for this risk, which include small-firm idiosyncratic volatility.

Keywords: Idiosyncratic risk; entrepreneurial risk; small firms.

JEL Classifications: G10, G12

1. Introduction

It is common practice to divide the risk of equity returns into two parts. *Systematic risk* is tied to the market return or other factors common across stocks, while *nonsystematic* or *idiosyncratic risk* is the remaining risk. Long ago, Markowitz (1952) demonstrated that optimal portfolios are constructed to eliminate idiosyncratic risk, while Sharpe (1964) taught us to expect compensation for bearing systematic risk and no compensation for idiosyncractic risk, provided that investors are well diversified. Furthermore, the intertemporal capital asset pricing model (ICAPM) of Merton (1973a) predicts a positive relation between expected market returns and systematic risk in a time series.

Authors since then have studied idiosyncratic volatility. Campbell *et al.* (2001) show that the idiosyncratic volatility of the average stock rose more than three times between 1962 and 1997. Wei and Zhang (2006) attribute much of this increase to a shift in the fundamental risks of public firms, and especially newly listed firms, where risk is measured by the volatility of book return on equity. Malkiel and Xu (2003) suggest instead that the increase in idiosyncratic volatility is the result of changes in institutional ownership and expected earnings growth. Pastor and Veronesi (2003) find that idiosyncratic volatility is inversely related to firm age, while Ang *et al.* (2006) demonstrate that expected returns and idiosyncratic volatility are inversely related in the cross-section.

Goyal and Santa-Clara (2003) study predictive regressions. Projecting stock market index returns jointly on past variances of index returns and value-weighted averages of total risk, they find positive and significant coefficients on the latter and negative coefficients on the former.[1] One interpretation of their result is that idiosyncratic risk is priced, a result that is unexpected given Merton's (1973a) ICAPM. Bali *et al.* (2005) reexamine the predictive regressions of Goyal and Santa-Clara (2003), comparing equal- and value-weighted averages of idiosyncratic volatility as regressors. They conclude that the market index return predictability is largely due to a liquidity premium and to small NASDAQ stocks, and that the predictive relation is much weaker or nonexistent in recent data.

This paper continues the work on idiosyncratic volatility and predictability of returns. We address two primary issues. First, we again examine the predictability of returns of portfolios using averages of idiosyncratic volatility. We extend the work of Goyal and Santa-Clara (2003) and Bali *et al.* (2005) in grouping stocks by market capitalization of equity into two *size portfolios*, namely a *small-firm* and a *big-firm* portfolio, and into *size-decile*

[1] Cross-sectional averages of individual stock return variances are easily calculated and, more importantly, are good proxies for idiosyncratic risk. This is because the lion's share of the variation in total volatility is due to variation in idiosyncratic volatility.

portfolios. We ask whether small-firm returns are predictable using either their own idiosyncratic volatility or that of big firms, and the same for big-firm returns. We support the idea that the idiosyncratic volatilities of small firms, but not those of big firms, predict future portfolio returns. Importantly, we demonstrate that the small-firm idiosyncratic volatility (*SMALL*) is a significant predictor of both big- and small-firm returns. Furthermore, *SMA LL* has predictive power in isolation and jointly with instrumental variables, that is, instruments for the business cycle and illiquidity, and *SMALL* has predictive power in recent data.

Our second task is to sort through several hypotheses and to suggest an economic explanation for the predictive power of *SMALL* for large- and small-firm returns. One hypothesis is that investors are imperfectly diversified, as in Merton (1987). His model is different from classic theories of asset pricing e.g., Sharpe (1964) and Merton (1973a) in that investors seek compensation for idiosyncratic risk. Merton (1987) suggests a positive relation between a stock's own returns and its idiosyncratic volatility of returns. A second hypothesis, considered also by Bali *et al.* (2005), is that investors demand compensation for illiquidity, as in Amihud and Mendelson (1986). Under this hypothesis, idiosyncratic volatility is a proxy for illiquidity. It is a positive predictor of realized return, which is the sum of the conditional expected return and an error, because it varies in a direct relation with illiquidity. Simply put, under this hypothesis the idiosyncratic risk of the average stock is high when the market is illiquid. A third hypothesis is that equity is priced as an option, as suggested by Black and Scholes (1973) and Merton (1974). Some have argued in this case that equity values and expected returns fluctuate with the volatility of the underlying assets and therefore the idiosyncratic risk of equity.

Our tests demonstrate that none of the first three hypotheses — *imperfect diversification, illiquidity* and *option hypotheses* — provides an adequate explanation of the predictive power of small-firm idiosyncratic risk. Each of these hypotheses implies that large-firm returns are predictable using the idiosyncratic volatility of large firms; the idiosyncratic volatility of small firms would be a useful predictor only if it were a proxy for that of large firms. Yet we find that the idiosyncratic volatilities of big firms do not predict big-firm returns, nor do they predict returns of any portfolios of firms grouped according to market capitalization. On the other hand, *SMALL* is special. It is a significant positive predictor of future returns of firms of all sizes, other than the returns of the 10% of largest firms. This is true when *SMALL* appears as a sole predictor, when it appears with systematic volatility or the idiosyncratic volatility of big firms, and when it appears with instruments.

We focus our attention on a fourth hypothesis, the existence of *background risk*, which is an idea attributed to a number of authors and summarized by Goyal and Santa-Clara (2003). Because investors hold assets other than stock — real estate, debt, nonpublic firms, and human capital — their total portfolio risk is not directly observable. If idiosyncratic volatility of equity is a proxy for the volatility of the total portfolio of the typical investor, equity values are low and expected returns are high when idiosyncratic risk is high.

Jaganathan and Wang (1996) study a specific case of background risk, generalizing the CAPM to incorporate human capital. In their data, equities with large covariances of returns with aggregate wages also have high average rates of return. Heaton and Lucas (2000) refine this idea, demonstrating the importance of *entrepreneurial risk*. Because U.S. households with proprietary business income are significant stockholders, stocks with high entrepreneurial risk exposure — measured by the covariance or beta of returns with proprietary business income — offer higher average returns in their cross-sectional study.

We estimate entrepreneurial risk using the mimicking portfolio technique first suggested by Breeden *et al.* (1989) and refined by Lamont (2001). We construct an *income portfolio*, which is a portfolio of stocks with monthly returns that best predict future proprietary business income. We then calculate a time series of monthly variances and covariances using daily returns. In this way, entrepreneurial risk is estimated as the covariance of stock returns with the income portfolio returns.

Our work extends that of Jaganathan and Wang (1996) and Heaton and Lucas (2000). Whereas they study the cross section, we examine the time series of expected returns. We find that both small- and big-firm returns are predictable using predetermined values of entrepreneurial risk, and the information in entrepreneurial risk is orthogonal to that in commonly used business cycle instruments. Furthermore, entrepreneurial risk competes strongly with *SMALL* as a predictor of returns; *SMALL* is a significant predictor of returns when entrepreneurial risk does not appear as a predictor, and it is an insignificant variable when entrepreneurial risk is added as a predictor. We conclude, therefore, that *SMALL* is a predictor of returns primarily because it is a proxy for entrepreneurial risk. We also conclude that entrepreneurial risk is an economically important determinant of variations in expected equity returns.

The remainder of the paper is organized as follows. Section 2 describes estimation and data. Section 3 describes projections of equity portfolio returns on *SMALL* and other measures of volatility, and it introduces our four hypotheses in detail. Section 4 presents the mimicking-portfolio model and describes the significance of entrepreneurial risk as a predictor of stock returns. It also discusses evidence supporting the entrepreneurial risk hypothesis. Section 5 concludes.

2. Methodology and Data

Equity return innovations tied to common factors or market returns create systematic risk. Nonsystematic or idiosyncratic risk arises from innovations that are specific to an equity issue or a small set of issues. One way to calculate systematic and idiosyncratic risks is to run a regression that projects equity returns on the returns of the market index, industry index or factors. We then calculate the volatility of the projection, which is systematic risk, and the volatility of the residual, which is orthogonal to the projection and is idiosyncratic risk. Intercepts and slopes, namely market or factor betas, are estimated in the regression.

We follow a second approach, which is to use the method of moments to directly estimate the idiosyncratic volatility. Consider the case of the market model first. The projection of the firms excess return on the market excess return provides

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it} = \alpha_i + \sigma_{im} R_{mt}/\sigma_m^2 + e_{it}, \tag{1}$$

with $E(e_{it}) = Cov(R_{mt}, e_{it}) = 0$; where R_{it} is the return of firm i in period t in excess of the risk free rate; $R_{mt} = \sum_i w_{it} R_{it}$ is the excess value-weighted market return, with w_{it} the firm i weight; $\sigma_{im} = Cov(R_{it}, R_{mt})$; and $\sigma_m^2 = Var(R_{mt})$. The projection also gives a decomposition of the variance:

$$\sigma_i^2 = \sigma_{im}^2/\sigma_m^2 + \sigma_{ie}^2. \tag{2}$$

We use sums of squares of daily returns to estimate variances σ_i^2 and σ_m^2, and sums of cross-products to estimate the covariance σ_{im}^2. Then the idiosyncratic volatility is estimated as

$$\hat{\sigma}_{ie}^2 = \hat{\sigma}_i^2 - \hat{\sigma}_{im}^2/\hat{\sigma}_m^2. \tag{3}$$

The estimate in Eq. (3) is equivalent to that in the regression strategy if the calculation of regression coefficients and the calculation of residuals use a single data set.

Estimation of the idiosyncratic volatility for multifactor and industry models is made similarly. A projection of the excess return on the factor excess returns gives

$$R_{it} = \alpha_i + \beta^T R_{Ft} + e_{it} = \alpha_i + C_{iF}^T V_F^{-1} R_{Ft} + e_{it}, \tag{4}$$

with $Cov(R_{Ft}, e_{it}) = 0$; where $R_{Ft} = \{R_{ft}\}$ is the vector of excess factor returns; $C_{iF} = Cov(R_{it}, R_{Ft})$ is the vector of covariances of stock i returns with the factors; and $V_F = Cov(R_{Ft}^T, R_{Ft})$ is the factor variance-covariance matrix. The idiosyncratic volatility is estimated as

$$\hat{\sigma}_{ie}^2 = \hat{\sigma}_i^2 - \hat{C}_{iF}^T \hat{V}_F^{-1} \hat{C}_{iF}, \tag{5}$$

where method of moments gives estimates on the right-hand side. The idiosyncratic volatility of the market model (3) is a special case of (5) with one factor, the market index.

Similarly, a simple industry model,

$$R_{it} = \alpha_i + \beta_i R_{mt} + \beta_{in} R_{nt} + e_{it}, \tag{6}$$

can be treated as a two-factor model where one of the factors (R_{nt}, the industry return) differs in the cross-section of stocks. For issue i, $R_{Ft} = (R_{mt}, R_{nt})$, so that C_{iF} is the two-element vector of covariances of stock i returns with market and industry returns.

Equations (3) and (5) are used to estimate the idiosyncratic volatility of individual firms.

2.1. *Data*

Monthly volatility estimates during the period August 1963 to December 2001 are calculated using daily excess returns. Idiosyncratic and systematic volatilities are derived using the market model, the industry model, and the three-factor model of Fama and French (1992). Thus, return data are required for individual equity issues, market indexes, industry indexes, and the Fama–French factors. Excess returns are differences of returns and the risk-free rate, which for each day of month t is the 30-day U.S. Treasury bill rate of return at the end of the prior month $t-1$, divided by the number of trading days in month t. Bill return data and daily equity returns come from the Center for Research in Security Prices (CRSP).

Ordinary common shares (CRSP share code 10 or 11) that trade on the NYSE, the AMEX, and the NASDAQ are included in our data. American Depository Receipts and unit trusts as well as other issues are excluded. Month-end prices and shares outstanding are taken from the monthly CRSP files. An issue is included in the sample for a given month if the prior month-end share price is higher than or equal to \$1; if an issue always traded under \$1 per share, it does not appear in the sample. Monthly shares outstanding and prices are used to calculate market capitalization weights, which in turn are used to construct portfolio returns and averages of volatility estimates.

There are 19,695 issues that pass the share code and price filters, and that have valid returns in the daily files and market capitalizations in the monthly files. There are 3872 issues in the daily returns file that do not pass our share code and price filters, and 201 firms that pass our filters (based upon the data from the monthly files) but have no daily returns.

Daily industry excess returns R_{nt} are value-weighted averages of excess returns of issues in an industry. All issues are grouped into one of 49 industries following Fama and French (1997) and according to the SIC classification. In addition, an excess value-weighted market return R_{mt} is constructed. Industry weights and market index weights are constant during month t and are calculated as relative market capitalizations as of the end of month $t-1$. The value-weighted market index that we construct has a correlation with the CRSP value-weighted index of 0.999.[2]

An additional filter is applied in the calculation of monthly volatility estimates. For month t, volatility is calculated for an issue only if in the CRSP monthly file there is a valid market capitalization at the ends of both months $t-1$ and t. This assures that an issue does not enter or exit the data during the month.

We form two size portfolios, *small* and *big*, where a *small firm* is an issue with market capitalization below the monthly median market capitalization of all NYSE issues, while other issues are *big firms*. We also use *size decile* portfolios, grouping firms into ten portfolios of equal numbers of stocks ranked by market capitalization. Returns of these portfolios are calculated using market-weighted averages. Averages of idiosyncratic volatilities of issues within the portfolios are also calculated.

Panel A of Table 1 reports the number of issues and market capitalization of issues in the size portfolios, while Panel B reports summary statistics of monthly excess returns of the market index and the size portfolios. Small-firm returns are substantially higher on average, are more volatile, and exhibit more predictability than big-firm returns. For example, the Ljung–Box statistic Q_{12} is reported as a test of the null hypothesis that the first 12 autocorrelations of excess returns are jointly zero. It has a 5% critical level of 21.03. For small-firm returns, the statistic is 47.36, casting significant doubt on the hypothesis of no predictability, while for big-firm returns the statistic is marginally significant, 21.51.

[2] The daily returns for the Fama and French (1992) small-minus-big (SMB) and high-minus-low (HML) factors are drawn from French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 1. Descriptive statistics of excess returns and volatility measures.

Panel A: Portfolio properties (August 1963–December 2001)

	Number of Issues	Percent of Issues	Percent of Market Cap	Median Size
Small firms	3642	78.61	10.75	35.88
Big firms	900	21.39	89.25	954.27

Panel B: Univariate statistics (August 1963–December 2001)

	Mean	Median	Std Dev	Min	Max	Skew	Kurt	AR_1	Q_{12}	ADF
R_m	0.553	0.865	4.487	−22.252	16.141	−0.468	4.948	0.048	23.384	−5.840
R_{Small}	0.933	1.380	5.991	−28.335	26.594	−0.426	5.310	0.188	47.356	−6.205
R_{Big}	0.520	0.820	4.410	−21.442	16.855	−0.393	4.855	0.023	21.507	−5.702
MKT_{Small}	0.135	0.074	0.258	0.007	4.528	11.528	186.823	0.211	61.144	−4.339
MKT_{Big}	0.170	0.101	0.319	0.007	5.951	13.363	237.554	0.225	81.045	−4.181
$SMALL$	1.808	1.448	1.104	0.767	7.811	2.609	11.202	0.866	4454.519	−2.248
BIG	0.563	0.460	0.460	0.164	4.174	3.978	23.402	0.836	3171.213	−3.627

Table 1. (*Continued*)

Panel C: Cross-correlations (August 1963–December 2001)

	R_m	R_{Small}	R_{Big}	MKT_{Small}	MKT_{Big}	$SMALL$	BIG
R_m	1						
R_{Small}	0.877	1					
R_{Big}	0.997	0.842	1				
MKT_{Small}	−0.326	−0.328	−0.317	1			
MKT_{Big}	−0.274	−0.287	−0.266	0.973	1		
$SMALL$	0.034	0.050	0.036	0.481	0.495	1	
BIG	−0.028	−0.035	−0.024	0.504	0.531	0.905	1

Note: Panel A reports descriptive statistics of small- and big-firm portfolios averaged across months. Small- and big-firm portfolios are firms grouped by market capitalization less and greater than the median market capitalization of NYSE issues, respectively, in each month of our sample. NYSE, AMEX and NASDAQ securities with share codes of 10 or 11 (ordinary shares) are included. R_m is the excess return of the value-weighted market index, while R_p for $p = Small$ and Big are the excess returns of value-weighted portfolios of small and big firms, respectively. Panels B and C report statistics for monthly returns and estimates of variances and covariances. $SMALL$, BIG and MKT_p, with $p = Small$ and Big, are calculated from sums of products of daily returns within months as described in Sec. 2. $SMALL$ and BIG are value-weighted averages of idiosyncratic variances of small- and big-firm returns, respectively, using the single index model. MKT_{Small} is the covariance of R_{Small} and R_m, and similarly for MKT_{Big}. All (co)variance estimates are multiplied by 10^2. AR_1 is the first-order autocorrelation. Q_{12} is the Ljung–Box statistic of the null of no autocorrelation up to order 12. The 5% critical value is 21.03. ADF is the augmented Dickey–Fuller statistic for unit root calculated with an intercept and 12 lags. The 5% critical value is −2.87.

2.2. *Estimation of volatility measures*

Systematic and idiosyncratic volatilities are regressors in our studies of returns, and they are initially calculated as estimates of return variances and covariances, or as cross-sectional averages of idiosyncratic variances. Return variances are estimated as sums over trading days within a month t,

$$\hat{\sigma}^2_{k,t} = \sum_{s=1}^{T} R^2_{k,s}, \tag{7}$$

and covariances of returns are obtained in a similar fashion,

$$\hat{\sigma}_{kj,t} = \sum_{s=1}^{T} R_{k,s} R_{j,s}, \tag{8}$$

where k and j are indexes for the individual equity or portfolio. For example, the estimate of covariance of returns on stock i and industry n for month t is $\hat{\sigma}_{in,t}$. We follow French *et al.* (1987), setting mean returns to zero in calculations of variances and covariances.[3] We discuss later alternative measures of volatility and alternatives to Eqs. (7) and (8).

Estimates of covariances of portfolio and market index returns are labeled

$$\mathrm{MKT}_{p,t} \equiv \hat{\sigma}_{pm,t}. \tag{9}$$

Averages (across firms) of idiosyncratic volatilities are calculated as

$$\hat{\sigma}^2_{e,t} = \sum_{i} w_{i,t} \hat{\sigma}^2_{i,et}, \tag{10}$$

where $w_{i,t}$ is the weight for firm i. We initially calculate $\hat{\sigma}^2_{ie,t}$ using the market model Eq. (3), while in Appendix A we consider alternatives that rely on Eq. (5). Regressors are constructed by averaging in Eq. (10) over large and small firms, creating *BIG* and *SMALL*, respectively. We calculate averages using market value weights $w_{i,t}$.

Sample distributions of variance estimates are positively skewed, violating the common assumption that regressors are normally distributed. We therefore consider alternative models of volatility. In particular, we calculate averages as in (10), except that we use the standard deviation $\hat{\sigma}_{i,et}$ and the logarithm $\ln(\hat{\sigma}^2_{i,et})$. We also examine the robustness of our results using alternative models of idiosyncratic volatility and sample variances/covariances. For example, we consider the Fama–French three-factor model

[3] Setting the mean to zero produces more accurate volatility forecasts than using the estimate of mean returns. See Figlewski (1997) for a survey of results.

[instead of the market model in Eq. (3)], and the French *et al.* (1987) adjustments for serial correlations and cross-serial correlation in returns [instead of the simple estimators in Eqs. (7) and (8)]. Some of these results are reported in the text whereas others appear in Appendix A.

3. Predicting Portfolio Returns

In this section, we study market and idiosyncratic volatilities as predictors of stock returns. We are motivated in part by Goyal and Santa-Clara (2003), who find a positive relation between the market index return and an equal-weighted average of lagged stock return variances (total risk). They also find that market risk has no predictive power for the return of a market return. Because total risk is divisible into market risk and idiosyncratic risk, and because the variation in their measure of total risk is primarily the result of variation in the average idiosyncratic risk, Goyal and Santa-Clara (2003) suggest that idiosyncratic risk may be priced in market returns. That is, they suspect that expected market returns vary directly with the level of the idiosyncratic risk of the average firm because investors desire compensation for that risk.

Our work in this section is motivated also by a desire to find an economic explanation for the predictive power of idiosyncratic risk. Later we consider four hypotheses — imperfect diversification, illiquidity, option pricing and entrepreneurial risk — and our empirical structure is chosen with these hypotheses in mind. In particular, we use averages of idiosyncratic volatilities instead of total volatility as a predictor. We average idiosyncratic risks across big and small firms, creating *BIG* and *SMALL*, respectively, as alternative predictors. We study the returns of portfolios of firms grouped according to market capitalization, rather than using a market index return as a sole variable to be predicted.[4]

Our ability to distinguish between the first three hypotheses on the one hand and the entrepreneurial risk hypothesis on the other is largely dependent on models where returns of one group of firms are projected on predetermined values of the average idiosyncratic risk of a second and disjoint group of firms. For example, we use *BIG* as a predictor of small-firm returns and *SMALL* as a predictor of big-firm returns. If we find that *SMALL* is a significant predictor of big-firm returns, or that *BIG* is a predictor of small-firm returns, then we cast doubt on the imperfect diversification, illiquidity,

[4] The sum of *MKT* and a value-weighted average of *BIG* and *SMALL* is equivalent to the total risk measure in Goyal and Santa-Clara (2003).

and option pricing hypotheses. Our reasoning appears later. For now, it is enough to say that these hypotheses would have *SMALL* a significant predictor of small-firm returns but not a predictor of big-firm returns, and similarly would have *BIG* a predictor of big-firm returns but not a predictor of small-firm returns.

The next section reports models using only measures of volatility as regressors, while models in Sec. 3.2 include instrumental variables representing business cycle and illiquidity. Section 3.3 summarizes our hypotheses and the inferences drawn from the models about the hypotheses.

3.1. *Projections of returns on volatility*

Our first set of models are projections of size and decile portfolio excess returns on lagged values of volatilities

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 V_{p,t-1} + \epsilon_t, \tag{11}$$

where $R_{p,t}$ is the excess return of a portfolio and $MKT_{p,t-1}$ and V_{t-1} are market and idiosyncratic risk of a portfolio, respectively. Table 2 reports the results. Panels A, C and E give projections of excess returns of small firms, while B, D, and F give big-firm excess returns. Panels A and B include all CRSP firms, while Panels C and D (E and F) have portfolios of only NYSE (NASDAQ) firms. Each panel contains three sets of four columns, and each set of columns contains the results for one of three alternative methods of calculating idiosyncratic volatility; we use averages of variance, standard deviation and log variance. Alternative sets of explanatory variables are reported in the rows of any one panel. In some rows we use a single variable, setting $b_1 = 0$, and in others we use pairs of volatilities as explanatory variables. Slope coefficients, Newey–West robust t-statistics, and adjusted R-squares ($\overline{R}^2$) are reported, while the intercepts a are not reported.[5] We also calculate bootstrap p-values, recognizing that the regressors are persistent and contemporaneously correlated with the market return; see Stambaugh (1999). The bootstrap p-values are not reported as they are similar to the Newey–West p-values. The sample period is August 1963 to December 2001.

First, note Panels A, C and E of Table 2. *SMALL* is a significant and positive predictor of small-firm returns. This is true when it is used in isolation and when it is used jointly with market risk *MKT*. For example, in panel A, $R_{p,t}$ is the value-weighted average return of all small CRSP firms,

[5] We report Newey–West t-statistics with six lags to correct for heteroskedasticity and autocorrelation in returns.

Table 2. Forecasts of size portfolios excess returns.

	Variance				Standard Deviation				ln Variance			
	MKT	*SMALL*	*BIG*	$\overline{R}^2$ (%)	*MKT*	*SMALL*	*BIG*	$\overline{R}^2$ (%)	*MKT*	*SMALL*	*BIG*	$\overline{R}^2$ (%)
Panel A: NYSE/AMEX/NASDAQ Small-firm excess return (August 1963–December 2001)												
(1)		0.738		1.65		0.236		1.69		0.017		1.59
		(2.839)				(3.130)				(3.128)		
(2)	−1.255	0.896		1.65	0.009	0.234		1.47	0.001	0.016		1.39
	(−1.126)	(2.926)			(0.041)	(2.344)			(0.274)	(2.377)		
Panel B: NYSE/AMEX/NASDAQ Big-firm excess return (August 1963–December 2001)												
(1)		0.332		0.62		0.118		0.77		0.009		0.86
		(1.766)				(2.117)				(2.352)		
(2)	−1.229	0.524		1.00	−0.075	0.145		0.62	0.000	0.009		0.64
	(−1.849)	(2.340)			(−0.428)	(1.807)			(−0.105)	(1.865)		
(3)	−0.724		0.504	0.00	0.076		0.001	−0.10	0.003		−0.002	0.06
	(−0.847)		(1.102)		(0.356)		(0.011)		(1.024)		(−0.388)	
Panel C: NYSE Small-firm excess return (August 1963–December 2001)												
(1)		1.233		1.50		0.323		1.58		0.019		1.54
		(2.556)				(2.740)				(2.729)		
(2)	−1.851	1.710		1.83	−0.056	0.349		1.38	0.001	0.018		1.33
	(−2.058)	(2.880)			(−0.252)	(2.227)			(0.199)	(2.274)		
Panel D: NYSE Big-firm excess return (August 1963–December 2001)												
(1)		0.484		0.41		0.132		0.47		0.008		0.47
		(1.200)				(1.409)				(1.507)		
(2)	−1.539	0.993		1.09	−0.121	0.202		0.39	−0.001	0.009		0.26
	(−2.476)	(2.220)			(−0.612)	(1.473)			(−0.206)	(1.247)		
(3)	−0.810		0.548	0.03	0.062		−0.025	−0.17	0.003		−0.003	0.00
	(−1.098)		(0.882)		(0.309)		(−0.186)		(1.071)		(−0.680)	

Table 2. (*Continued*)

	Variance				Standard Deviation				ln Variance			
	MKT	*SMALL*	*BIG*	$\overline{R}^2$ (%)	*MKT*	*SMALL*	*BIG*	$\overline{R}^2$ (%)	*MKT*	*SMALL*	*BIG*	$\overline{R}^2$ (%)
Panel E: NASDAQ Small-firm excess return (August 1963–December 2001)												
(1)		0.725		2.94		0.243		2.83		0.017		2.43
		(2.905)				(3.114)				(2.924)		
(2)	−1.117	0.894		2.92	−0.012	0.248		2.55	0.001	0.017		2.15
	(−0.894)	(3.069)			(−0.059)	(2.234)			(0.151)	(2.110)		
Panel F: NASDAQ Big-firm excess return (August 1963–December 2001)												
(1)		0.470		1.43		0.176		1.71		0.014		1.80
		(1.921)				(2.264)				(2.473)		
(2)	−1.472	0.793		1.94	−0.124	0.229		1.59	−0.001	0.015		1.53
	(−1.991)	(2.510)			(−0.782)	(2.053)			(−0.296)	(1.984)		
(3)	−0.439		0.507	−0.10	0.143		−0.011	0.08	0.005		−0.001	0.25
	(−0.356)		(0.643)		(0.523)		(−0.048)		(0.959)		(−0.143)	

Note: Estimates of the slope coefficients of

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 V_{p,t-1} + \epsilon_t,$$

are shown. R_p for $p =$ Small and Big are monthly excess returns of value-weighted portfolios of small and big firms, respectively. $V_{p,t-1}$ is alternatively represented by *SMALL* or *BIG*. *SMALL*, *BIG* and MKT_p, with $p =$ Small and Big, are calculated from sums of products of daily returns within months as described in Sec. 2. *SMALL* and *BIG* are value-weighted averages of idiosyncratic variances of small- and big-firm returns, respectively, using the single index model. MKT_{Small} is the covariance of R_{Small} with the market index return, and similarly for MKT_{Big} All (co)variance estimates are multiplied by 10^2. Newey–West t-statistics with six lags are in parentheses.

and in the first four columns of this panel is the case that *SMALL* is the average of idiosyncratic variances of all small firms. In row (1) of the panel is the case that market risk is excluded. Here in the first set of four columns, the estimated coefficient on *SMALL* is 0.738 with Newey–West t-statistic of 2.839. Similarly in row (2) is the case where market risk is included, and the values are 0.896 and 2.926, respectively.

The second and third sets of four columns in each panel of Table 2 are cases where *SMALL* is an average of standard deviation and log variance, respectively, instead of an average of idiosyncratic variances. In each set of columns of Panel A, *SMALL* is a positive and significant predictor of small-firm returns. For example, in row (1) in the right-most columns *SMALL* is an average of logarithms of variance; its coefficient is 0.017 with a Newey–West t-statistic of 3.128. Similarly, *SMALL* is a positive and significant predictor across all the columns in Panels C and E, where $R_{p,t}$ is the average of small NYSE firms and small NASDAQ firms, respectively. The evidence is clear. The idiosyncratic volatility of small firms is a strong predictor of small-firm returns. This is true for both NYSE and NASDAQ small firms.

Now consider Panels B, D and F of Table 2, which report projections of the returns of big firms $R_{p,t}$ on lagged measures of volatility, and particularly consider the last row of each panel. These rows give projections of big-firm returns on *MKT* and *BIG*. We see that neither market risk *MKT* nor the average idiosyncratic risk of big firms *BIG* is a significant predictor of big-firm returns. This is true in Panel B using all CRSP firms, and in Panels D and F using NYSE and NASDAQ firms, respectively. This also is true across the three sets of columns giving the results for alternative measures of idiosyncratic risk. All of the estimated coefficients on *BIG* are insignificant, and many are negative or near zero. This evidence also is clear. The idiosyncratic volatility of big firms is not a predictor of big-firm returns.[6]

The first two rows of Panels B, D and F of Table 2 give projections of big-firm returns on the idiosyncratic volatility of small firms. In each of these panels, *SMALL* is used in isolation in row (1), and jointly with market risk *MKT* in row (2). In each case *SMALL* is a positive predictor of excess returns of big firms, but levels of significance are mixed across cases. For example, used in isolation the idiosyncratic volatility of small firms is a significant

[6] Further evidence that supports this conclusion but is not reported here includes regressions of big-firm returns on *BIG* and *SMALL* jointly. In these regressions, *SMALL* is significant in many cases whereas *BIG* is insignificant in all cases.

predictor of big-firm returns (at the 5% level) in the middle and right columns of Panels B and F; in these columns *SMALL* is an average of standard deviations and log variances, respectively. However, it is insignificant at the 5% level in the left-hand columns of these panels, where *SMALL* is an average of variances. *SMALL* also is insignificant in each of the three sets of columns in Panel D, where $R_{p,t}$ is the return on a portfolio of big NYSE firms only. The evidence in Panels B, D and F of Table 2 leaves room for doubt about the degree to which *SMALL* contains information for returns of big and midsize firms.

The excess returns of the big-firm portfolio are market-weighted average returns. The very biggest firms are the principal determinants of the portfolio returns. For this reason, we rank firms by market capitalization of all NYSE issues and form 10 decile portfolios, where firms are grouped in portfolios by market capitalization rank and where portfolio 1 (10) includes only the smallest (largest) firms. We create the portfolios twice, once using all CRSP firms and once using only NYSE firms. For each portfolio we run a univariate regression, which is model (11) with $b_1 = 0$ and where $V_{t-1} =$*SMALL* calculated as an average of variances. Row (1) in each of Panels A and B of Table 3 reports the estimated slope coefficients b_2, with Newey–West t-statistics in parentheses for two cases — all CRSP firms and NYSE firms — respectively.

Uniformly across the columns of Panel A of Table 3, *SMALL* is a positive and significant predictor of decile portfolio returns. In Panel B of Table 3, *SMALL* is a significant predictor of 8 of the 10 decile portfolios at the 5% level, and it is a marginally significant predictor of the returns of one of the other two portfolios. The one exception is portfolio 10, which contains the largest of the NYSE firms. Given that NYSE firms are on average larger than AMEX and NASDAQ firms, the evidence strongly supports the conclusion that *SMALL* is a significant predictor of all but a small minority of the largest firms. Row (2) includes both market risk and *SMALL* jointly as predictors; *SMALL* is always significant and positive.

Summarizing, Tables 2 and 3 lead us to several conclusions. First, *SMALL* is special and *BIG* is not. The idiosyncratic risk of small firms contains significant information for future stock returns, and particularly returns of medium- and small-sized firms, and the idiosyncratic risk of big firms *BIG* has no predictive information for firms of any size, big or small. Second, *SMALL* is not a proxy for market risk. When we include both market risk and *SMALL* jointly as predictors, *SMALL* is generally significant and positive.

Table 3. Forecasts of size decile portfolios excess returns.

		Small	2	3	4	5	6	7	8	9	Big
Panel A: NYSE/AMEX/NASDAQ (August 1963–December 2001)											
(1)	*SMALL* ($b_1 = 0$)	0.528	0.565	0.460	0.373	0.373	0.374	0.387	0.425	0.374	0.285
		(2.444)	(3.064)	(2.821)	(2.362)	(2.498)	(2.785)	(2.891)	(3.413)	(3.260)	(2.303)
(2)	*SMALL*	0.655	0.642	0.528	0.439	0.421	0.419	0.448	0.483	0.460	0.421
		(2.826)	(3.137)	(2.877)	(2.381)	(2.414)	(2.619)	(2.908)	(3.339)	(3.507)	(3.323)
Panel B: NYSE (August 1963–December 2001)											
(1)	*SMALL* ($b_1 = 0$)	1.495	1.191	1.203	0.865	1.031	1.038	0.844	0.871	0.841	0.316
		(2.339)	(2.296)	(2.441)	(1.927)	(2.661)	(2.496)	(1.998)	(2.312)	(2.101)	(0.755)
(2)	*SMALL*	2.619	1.887	1.881	1.430	1.578	1.627	1.382	1.417	1.412	0.981
		(3.301)	(2.724)	(3.009)	(2.328)	(3.128)	(2.865)	(2.414)	(2.799)	(2.832)	(2.099)

Note: Estimates of the slope coefficient b_2 of

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 SMALL_{t-1} + \epsilon_t,$$

are shown. R_p for $p = 1, \ldots, 10$ are monthly excess returns of value-weighted portfolios of firms grouped into deciles according to market capitalization. Panels A and B report results using all NYSE, AMEX and NASDAQ firms and all NYSE firms, respectively. *SMALL* and MKT_p, $p = 1, \ldots, 10$, are calculated from sums of products of daily returns within months as described in Sec. 2. *SMALL* is the value-weighted average of idiosyncratic variances of small firms, using the single index model. MKT_p is the covariance of R_p with the market index return. All (co)variance estimates are multiplied by 10^2. Row 1 of each panel sets $b_1 = 0$. Newey–West t-statistics with six lags are in parentheses.

3.2. *Projections with instrumental variables*

It is natural to be curious about the robustness of the results in the preceding section. Perhaps *SMALL* is a significant predictor of excess stock returns because it is a proxy for factors representing changes in the investment opportunity set. The literature documents predictable components of returns using a large number of business-cycle variables.[7] We also know that high-frequency returns, such as daily or weekly returns, and particularly those of small firms, are autocorrelated and cross-autocorrelated with past market returns, and these predictable components are the result of infrequent trading. Perhaps *SMALL* is a proxy for the frequency of trading and this explains our results using monthly excess returns. It is also possible that *SMALL* is a significant predictor of returns because it is a proxy for illiquidity. In this section, we investigate these alternative explanations.

We study projections of excess returns of size and decile portfolios on instrumental variables in addition to measures of volatility. We use the model

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2\, V_{p,t-1} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t, \tag{12}$$

where $MKT_{p,t-1}$ and $V_{p,t-1}$ are measures of market and idiosyncratic risk defined previously, and $X_{t-1}^{(j)}$ are instrumental variables.

Panels A and B of Table 4 report results for small- and big-firm returns, respectively. Projections in the first three rows of each panel use six instruments. These include five business-cycle variables, which are lagged values of dividend-price ratio DP_{t-1}, relative Treasury bill rate RTB_{t-1}, term spread TS_{t-1}, default spread DS_{t-1}, and consumption-wealth ratio CAY_{t-1}. DP_{t-1} is the sum of dividends over the 12 months ending with month $t-1$ divided by the CRSP value-weighted index at the end of month $t-1$. RTB_{t-1} is the difference between the one-month Treasury bill rate at month end and the average of rates over the 12 prior months. TS_{t-1} is the difference between the yield on long-term government bonds and the yield on one-month Treasury bills at the end of month $t-1$. Each of these variables is calculated from the Ibbotson Associates Stocks, Bonds, Bills and Inflation yearbook data. DS_{t-1} is the difference in yields on BAA and AAA-rated corporate bonds in the FRED database at the St. Louis Federal Reserve Bank, while CAY_{t-1} is the consumption-wealth ratio of Lettau and Ludvigson (2001).

[7] See Keim and Stambaugh (1986), Fama and French (1988), Fama and French (1989) and Fama (1990) as examples.

Table 4. Forecasts of size portfolios excess returns using business cycle variables.

	MKT	*SMALL*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	*IL*	$\overline{R}^2$ (%)
Panel A: NYSE/AMEX/NASDAQ Small-firm excess return (August 1963–December 2001)										
(1)			0.002	−5.092	−0.857	3.614	3.614	0.109		8.64
			(0.183)	(−2.037)	(−3.971)	(3.539)	(3.539)	(1.700)		
(2)		1.514	0.044	−4.059	−1.042	2.975	2.975	0.147		12.24
		(4.289)	(2.592)	(−1.628)	(−4.936)	(2.959)	(2.959)	(2.092)		
(3)	−1.905	1.867	0.052	−4.276	−1.088	2.992	2.992	0.114		12.42
	(−1.540)	(4.406)	(2.761)	(−1.648)	(−5.000)	(2.923)	(2.923)	(1.620)		
(4)			0.008	−5.081	−0.931	3.680	0.740	0.105	−0.002	8.49
			(0.474)	(−2.040)	(−3.484)	(3.593)	(3.082)	(1.637)	(−0.569)	
(5)	−2.269	2.123	0.083	−4.144	−1.420	3.181	0.574	0.098	−0.010	12.97
	(−1.673)	(4.975)	(3.684)	(−1.607)	(−4.987)	(3.275)	(2.599)	(1.390)	(−1.928)	

Table 4. (*Continued*)

	MKT	*SMALL*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	*IL*	$\overline{R}^2$ (%)
Panel B: NYSE/AMEX/NASDAQ Big-firm excess return (August 1963–December 2001)										
(1)			−0.009	−3.288	−0.545	2.866	2.866	−0.059		6.92
			(−0.873)	(−1.568)	(−3.773)	(4.168)	(4.168)	(−1.116)		
(2)		0.538	0.006	−2.921	−0.611	2.639	2.639	−0.046		7.60
		(2.085)	(0.460)	(−1.390)	(−4.067)	(3.688)	(3.688)	(−0.812)		
(3)	−2.041	0.999	0.015	−3.355	−0.645	2.620	2.620	−0.082		8.69
	(−2.894)	(3.079)	(0.981)	(−1.495)	(−4.064)	(3.545)	(3.545)	(−1.654)		
(4)			0.008	−3.023	−0.729	2.927	0.667	−0.060	−0.004	6.97
			(0.500)	(−1.458)	(−3.407)	(4.280)	(3.697)	(−1.130)	(−1.187)	
(5)	−2.045	0.970	0.028	−3.166	−0.787	2.680	0.621	−0.083	−0.003	8.64
	(−2.808)	(2.947)	(1.421)	(−1.424)	(−3.354)	(3.610)	(3.320)	(−1.694)	(−0.869)	

Note: Estimates of slope coefficients of

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 SMALL_{t-1} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t,$$

are shown. R_p for $p =$ Small and Big are monthly excess returns of value-weighted portfolios of firms grouped by market capitalization less and greater than the median market capitalization of NYSE issues, respectively, in each month of our sample. *SMALL* and MKT_p, with $p =$ Small and Big, are calculated from sums of products of daily returns within months as described in Sec. 2. *SMALL* is the value-weighted average of idiosyncratic variances of small-firm returns using the single index model. MKT_{Small} is the covariance of R_{Small} with the market index return, and similarly for MKT_{Big} All (co)variance estimates are multiplied by 10^2. The $X_{t-1}^{(j)}$ are the following predetermined variables. *DP* is the logged dividend price ratio calculated as the difference between the log of prior 12 month dividends and the log of the month $t-1$ price index of the CRSP value-weighted index. *RTB* is the the difference between T-bill yield and its 12-month moving average. *TS* is the difference between the yield on long-term government bonds and 1 month T-bill rate of return. *DS* is the difference between the yield on BAA-rated and AAA-rated corporate bonds. *CAY* is the trend deviations in the log consumption-wealth ratio in Lettau and Ludvigson (2001). $R_{m,t-1}$ is the lagged value-weighted market excess return. IL_p the value-weighted average of the illiquidity measure in Amihud (2002), averaged over small- and big-firms, respectively. Newey–West t-statistics with six lags are in parentheses.

Each of the business-cycle instruments is a known quantity at the beginning of month t.[8]

The sixth instrumental variable in the first three rows of the panels in Table 4 is the lagged return on the CRSP value-weighted index $R_{m,t-1}$. Based on the arguments of Boudoukh *et al.* (1994), we expect the coefficient on $R_{m,t-1}$ to be positive, and to be larger for small firms than for big firms, because small firms trade less frequently and have more volatile returns. If $SMALL$ is a proxy for infrequent trading, then by including $R_{m,t-1}$ we expect $SMALL$ to be insignificant in the regression. Similarly, if $SMALL$ is proxy for the business cycle instruments, then again we expect $SMALL$ to be insignificant in all rows of the table.

Again, Panels A and B of Table 4 report results for small- and big-firm returns, respectively. In each panel, row (1) includes only the six instrumental variables; volatility measures do not appear. Row (1) is our base case and first point of comparison for other rows in the same panel. In rows (2) and (3) of each panel, $SMALL$ appears in isolation and jointly with MKT, respectively. If $SMALL$ is important for reasons other than as a proxy for the business cycle or for infrequent trading, then it should be a significant predictor in rows (2) and (3) where it is added to the base case. We should also see a larger explanatory power than in row (1). Alternatively, if $SMALL$ is simply a proxy for the instruments, it is likely to be insignificant in rows (2) and (3), and the adjusted R-squares ($\overline{R}^2$) of these rows should be similar to that of row (1).

Consider row (1) of each of Panels A and B. The six instruments jointly have considerable explanatory power. For small-firm returns in Panel A, the relative Treasury bill yield, term spread, default spread, and consumption-wealth ratio are significant at the 5% level, and the $\overline{R}^2$ is above 8%. The case of large firms in Panel B is similar, except that RTB is not significant and the $\overline{R}^2$ is about 7%.

Now examine rows (2) and (3). $SMALL$ is significant at the 5% level, and $\overline{R}^2$ is much larger than in row (1). This is true in both panels, but particularly noteworthy is a comparison of rows (1) and (2) in Panel A of Table 4. In this study of small-firm returns, $\overline{R}^2$ is 8.64% without $SMALL$ and 12.24% with $SMALL$.

In summary, the first three rows of Panels A and B of Table 4 are evidence that $SMALL$ is more than a proxy for our business-cycle variables or a proxy for infrequent trading.

[8] CAY is quarterly; we use quarterly observations for each month of a quarter. CAY is obtained from Lettau's website, http://pages.stern.nyu.edu/~mlettau.

Before considering rows (4) and (5), look at Table 5. It reports projections of size decile portfolio returns on the same regressors that appear in the first three rows of Table 4. The table is constructed similarly to Table 3. Only the coefficients on *SMALL* are reported. Panels A and B report results for decile portfolios that include all NYSE/AMEX/NASDAQ firms and only NYSE firms, respectively. Rows (1) and (2) are cases where *MKT* is excluded and included respectively as a regressor.

The results in Table 5 for size decile portfolios support our conclusions drawn from Table 4 for size portfolio returns. Importantly, *SMALL* is a significant predictor at the 5% level of the excess returns of all decile portfolios with the exception of the portfolio of the largest 10% of firms. Again, we conclude *SMALL* is more than a proxy for the business-cycle instruments and measures of infrequent trading.

As a final point in this vein, compare the coefficients in Panel A of Table 5 to the corresponding values of Panel A of Table 3, and similarly compare coefficients in Panel B. Again, Panel A of each table reports results for all CRSP firms while Panel B uses only NYSE firms. The reported coefficients are those on *SMALL*, and the difference across the tables is that business-cycle instruments appear as regressors in Table 5 but not in Table 3. We see that the coefficients and robust t-statistics in Table 5 are considerably larger than those in Table 3. This implies that *SMALL* contains information that is orthogonal to future stock return R_{pt} but is correlated with information in the business-cycle instruments as a group. As a result, we must be careful interpreting the magnitude of both the coefficients and t-statistics for *SMALL* in predictive regressions when controls are present. What appears as significant information in *SMALL* that is useful in predicting returns may instead be information that predicts the instruments. We return to this point later in the discussion of entrepreneurial risk.

We next add to our set of instrumental variables the measure of illiquidity suggested in Amihud (2002). For each stock i, we average across days in each month t to get

$$IL_{i,t} = \frac{1}{T}\sum_{s=1}^{T}\frac{|R_{i,s}|}{VOLD_{i,s}}, \tag{13}$$

where $|R_{i,s}|$ is the absolute return on day s, $VOLD_{i,s}$ is the dollar trading volume on day s, and T is the number of trading days in the month. $IL_{i,t}$ is the average absolute daily return per unit of dollar volume and is a direct measure of price pressure and therefore illiquidity of stock i. Our regressors

Table 5. Forecasts of size decile portfolios excess returns.

		Small	2	3	4	5	6	7	8	9	Big
Panel A: NYSE/AMEX/NASDAQ (August 1963–December 2001)											
(1)	*SMALL* ($b_1 = 0$)	0.903	0.847	0.794	0.663	0.620	0.640	0.551	0.619	0.498	0.241
		(3.567)	(3.666)	(3.847)	(3.353)	(3.276)	(3.501)	(3.082)	(3.174)	(2.755)	(1.222)
(2)	*SMALL*	1.011	0.931	0.897	0.771	0.709	0.743	0.678	0.788	0.695	0.508
		(3.434)	(3.325)	(3.566)	(3.050)	(2.961)	(3.161)	(2.907)	(3.339)	(3.074)	(2.251)
Panel B: NYSE (August 1963–December 2001)											
(1)	*SMALL* ($b_1 = 0$)	1.976	1.528	1.457	1.193	1.224	1.230	0.992	1.048	0.939	0.179
		(3.072)	(2.755)	(2.848)	(2.460)	(2.626)	(2.729)	(2.406)	(2.474)	(2.126)	(0.379)
(2)	*SMALL*	2.718	2.037	1.924	1.612	1.613	1.756	1.498	1.639	1.578	0.952
		(3.534)	(2.983)	(2.970)	(2.700)	(2.668)	(2.997)	(2.643)	(2.862)	(2.923)	(1.782)

Note: Estimates of the slope coefficient b_2 of

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 SMALL_{t-1} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t$$

are shown. R_p for $p = 1, \ldots, 10$ are monthly excess returns of value-weighted portfolios of firms grouped into deciles according to market capitalization. Panels A and B report results using all NYSE, AMEX and NASDAQ firms and all NYSE firms, respectively. $SMALL$ and MKT_p, $p = 1, \ldots, 10$, are calculated from sums of products of daily returns within months as described in Sec. 2. $SMALL$ is the value-weighted average of idiosyncratic variances of small firms, using the single index model. MKT_p is the covariance of R_p with the market index return. All (co)variance estimates are multiplied by 10^2. The $X_{t-1}^{(j)}$ are the following predetermined variables. DP is the logged dividend price ratio calculated as the difference between the log of prior 12 month dividends and the log of the month $t-1$ price index of the CRSP value-weighted index. RTB is the difference between T-bill yield and its 12-month moving average. TS is the difference between the yield on long-term government bonds and 1 month T-bill rate of return. DS is the difference between the yield on BAA-rated and AAA-rated corporate bonds. CAY is the trend deviations in the log consumption-wealth ratio in Lettau and Ludvigson (2001). $R_{m,t-1}$ is the lagged value-weighted market excess return. IL_p the value-weighted average of the illiquidity measure in Amihud (2002), averaged over small- and big-firms, respectively. Row 1 of each panel sets $b_1 = 0$. Newey–West t-statistics with six lags are in parentheses.

are value-weighted averages

$$IL_{p,t} = \sum_i w_{i,t} IL_{i,t}, \tag{14}$$

where $p = \mathit{Small}$ and Big.[9]

Amihud (2002) uses Eq. (14) to obtain a marketwide measure of illiquidity; he calculates an equal-weighted average of the $IL_{k,t}$ across all NYSE stocks. He projects excess returns $R_{p,t}$ of an equal-weighted market index and size-based portfolios on lagged values of $IL_{p,t-1}$. He finds that illiquidity is a significant and positive predictor of index returns, and he interprets the results as evidence of an illiquidity premium — expected stock returns are high when illiquidity is high.

We use $IL_{p,t-1}$ as one of the instruments, estimating model (12) separately for small- and big-firm returns. Therefore, we calculate two measures of illiquidity, IL_{Small} and IL_{Big}, averaging in Eq. (14) across small and big firms, respectively.[10] We use IL_{Small} as a predictor of small-firm returns, and similarly IL_{Big} as a predictor of big-firm returns. If $IL_{p,t-1}$ is a good measure of illiquidity and if there is an illiquidity premium, the coefficients on $IL_{p,t-1}$ in the regressions should be positive for both small and big firms. On the other hand, it is possible that there are differences across small and big firms. For example, we might believe only small-firm returns exhibit liquidity premiums, in which case we expect to see that IL_{Small} is a significant predictor of excess returns of small firms, while IL_{Big} is insignificant in projections of big-firm returns. Importantly, this reasoning does not suggest that either IL_{Small} should predict big-firm returns or IL_{Big} should predict small-firm returns.

Rows (4) and (5) of Panels A and B of Table 4 give projections of excess returns of portfolios of small and big firms, respectively, on MKT, $SMALL$ and instrumental variables, including illiquidity $IL_{p,t-1}$. IL_{Small} appears in Panel A, and IL_{Big} is used in Panel B. Row (4) of each panel includes the full set of instruments and does not include volatility as a regressor, while row (5) includes $SMALL$ and MKT as regressors in addition to the full set of instruments in row (4).

[9] We use value-weighted averages of IL, as we do in calculating averages of idiosyncratic volatility. We reach the same conclusions using equal-weighted averages of IL.
[10] Following Amihud (2002), we use only NYSE-traded stocks from the CRSP daily stock file. NASDAQ data are available from CRSP beginning only in 1982, and NASDAQ volumes include interdealer trades, unlike the NYSE volumes.

Row (4) of each panel is a new base case and point of comparison for the study of *SMALL* as a proxy for illiquidity. If *SMALL* is important for reasons other than a proxy for liquidity (and other instrumental variables), then it should be significant in row (5), and the explanatory power shown in that row should be greater than in row (4). Alternatively, if *SMALL* adds no information to the prediction of excess returns, then it should be insignificant in row (5), and the $\overline{R}^2$ should be little different between rows (4) and (5).

The evidence in Table 4 does not support the idea that *SMALL* is a proxy for illiquidity. *SMALL* is a positive and significant variable in row (5) of both Panels A and B. The Newey–West t-statistics are 4.975 and 2.947, respectively. Consequently, we believe that *SMALL* is a positive and significant predictor of stock returns, and it is more than a proxy for business-cycle instruments and measures of illiquidity.

In a recent challenge to Goyal and Santa-Clara (2003), Bali *et al.* (2005) project equity portfolio returns on measures of idiosyncratic volatility. They use many of the same regressors as appear here, and in particular they study illiquidity as a regressor. Importantly, they give a negative answer to the question posed in their provocative title "Does Idiosyncratic Risk Really Matter?" because their measures of idiosyncratic volatility are not significant predictors of returns when illiquidity is included as a regressor. Our results seem to contradict their results. However, a direct comparison cannot be made for a variety of reasons.

One difference is that we use *SMALL* and *BIG* as regressors whereas they use averages of idiosyncratic volatility across all firms. This allows us to demonstrate that there is significant information in the idiosyncratic risks of small firms and none in that of big firms. Another difference is that we forecast returns of size portfolios and size decile portfolios. This allows us to show that the returns of all but the largest 10% of firms are predictable using *SMALL*.

The third and most important difference is that we consider only predetermined variables as regressors, whereas Bali *et al.* (2005) include a contemporaneous variable in their projections. They decompose illiquidity into two components – expected and unexpected — using a time series regression. They then project returns R_t jointly on the two components together with measures of idiosyncratic volatility. In these regressions, expected illiquidity is a predetermined variable known in month $t-1$, while unexpected illiquidity is a contemporaneous variable known in month t but not before. Idiosyncratic volatility in their model (as in our model) is a predetermined variable known in month $t-1$. The consequence is that unexpected illiquidity as a contemporaneous regressor soaks up the information in idiosyncratic volatility for month $t-1$, and the latter appears to be an insignificant predictor of month t returns R_t. Because the regressions of Bali *et al.* (2005) where unexpected illiquidity appears are not predictive models, they cannot be used to judge if idiosyncratic risk matters. Their results cannot be compared to ours.

As a final analysis of this section, we judge the importance of illiquidity as an explanatory variable for the time series of expected stock returns. Compare rows (1) and (4) of Panels A and B of Table 4; volatility measures do not appear as regressors in these rows. If there is a liquidity premium for small firms, IL_{Small} should be significant in row (4) of Panel A, and the explanatory power of that regression should be significantly greater than the explanatory power in row (1). A similar comparison should be seen in rows (1) and (4) of Panel B for big-firm returns.

The evidence in Panel A of Table 4 does not support the idea of a liquidity premium in the small-firm returns. In row (4) of Panel A the t-statistic for the coefficient on IL_{Small} is less than -0.57 and is insignificant at the 5% level; the $\overline{R}^2$ of row (4) is 8.49% compared to 8.64% in row (1). Similarly, a comparison of rows (1) and (4) of Panel B does not support a liquidity premium in big-firm returns. Row (4) of that panel shows that IL_{Big} is not a significant predictor of big-firm returns.

3.3. *Additional models*

The primary thrust of our work thus far is to address these questions: Does the idiosyncratic volatility of small firms *SMALL* have predictive power for stock returns? Is *SMALL* a proxy for commonly used business-cycle instruments or alternatively a proxy for illiquidity?

Because it is natural to ask whether our results and our answers to these questions are robust to particular choices that we have made, in either our choice of data, our methods of calculations, or our choice of regressors to include in our projections, Table A.1 appears in Appendix A along with a brief discussion. This table reports results where we tweak our constructions in a variety of ways. We use alternative calculations of monthly variances and covariances from daily data. We break the sample into subperiods. We use projections that forecast stock-index futures returns. We use alternative measures of liquidity, namely turnover and dollar volume. We add seasonal dummies and alternative risk factors as regressors.

None of these results seriously challenge our prior conclusions. We must find an explanation for *SMALL* as a predictor of returns.

3.4. *Four hypotheses*

The analysis of the preceding sections tells us that *SMALL* is a significant predictor of future stock returns, but it is not a proxy for commonly used business-cycle instruments, nor is it a proxy for illiquidity. In this section we offer economic arguments suggesting why *SMALL* contains predictive information. We consider four hypotheses, which are the *imperfect diversification, liquidity*, *option* and *entrepreneurial risk* hypotheses.

3.4.1. *Imperfect diversification hypothesis*

Under the *imperfect diversification hypothesis*, investors are not perfectly diversified as in Merton (1987), their total portfolio risk increases with idiosyncratic risk, and investors seek compensation for this risk. If this hypothesis applies to all firms, the expected rates of return of big firms are positively related to the idiosyncratic risk of big firms (meaning that *BIG* should be significant in Panel B of Table 2). Similarly, for any size portfolio, return should be positively related to the average of idiosyncratic volatilities of the same collection of firms. The evidence does not support this hypothesis. For example, big-firm returns increase with *SMALL* but are insignificantly related to *BIG* in Tables 2 and 4.

Perhaps investors in small firms are less well diversified than investors in big firms. For example, Ofek and Richardson (2003) show that large numbers of insiders sell out their positions in the years following IPOs, and IPO firms tend to be smaller than older firms. In this case, we expect that small-firm returns are positively related to *SMALL*, while big-firm returns are insignificantly related to *BIG*. We see evidence supporting this idea in Table 2.

The imperfect diversification hypothesis, however, does not explain why *SMALL* is a significant and positive predictor of big-firm returns. More important, it does not explain why *SMALL* dominates *BIG* as a predictor of big-firm returns, as is shown by rows (2) and (3) of Panels B, D and F in Table 2.

We do not believe that the imperfect diversification hypothesis provides a good explanation of either our results or the evidence in the existing literature that idiosyncratic volatility predicts stock returns. We do not believe that idiosyncratic risk is priced.

3.4.2. *Liquidity hypothesis*

Grossman and Stiglitz (1980) and Kyle (1989) predict that the price impact of market orders to trade stocks varies inversely with the depth of the market. As a result, volatility is inversely related to liquidity, and it is a direct proxy for illiquidity. For example, we expect high idiosyncratic volatility when the bid-ask spread is wide. Furthermore, Amihud and Mendelson (1986) predict that expected returns and liquidity are inversely related.

The implications of the *liquidity hypothesis* for our data are much like those of the imperfect diversification hypothesis. For example, we should see that *BIG* is a positive and significant predictor of big-firm returns; that is, *BIG* should be significant in Panels B, D, and F of Table 2. We should also see that *BIG* is a proxy for one or more measures of illiquidity, and that when we add these measures to the regression *BIG* is no longer significant. We should also see that *BIG* dominates *SMALL* as a predictor of big-firm returns.

As we note in the preceding section, the evidence is contrary to this hypothesis. *BIG* is not a significant predictor of big-firm returns. *BIG* is not dominated as a predictive variable by measures of illiquidity. *SMALL* is a significant and positive predictor of portfolio returns, and particularly to our arguments, *SMALL* is a significant predictor of big-firm returns.

We do not believe the liquidity hypothesis is a good explanation of our results or the evidence in the existing literature about the behavior of aggregate measures of idiosyncratic volatility.

3.4.3. *Option hypothesis*

Black and Scholes (1973) and Merton (1973b) model the equity of a levered firm as a call option on the firms assets. An implication of this *option hypothesis* is that total equity volatility and expected rate of return covary in a positive fashion through time (because each is directly related to the degree of leverage). One is tempted to argue that these models imply that idiosyncratic volatility, which is a large part of total volatility for most firms, is a positive and significant predictor of stock returns.

There are two problems with this argument. The first is that the predictions are inconsistent with our evidence. The argument leads to the conclusion that *BIG* predicts big-firm returns, and it has nothing to say about *SMALL* as a predictor of big-firm returns. As already noted, the data contradict these predictions.

The second problem with the option hypothesis is a fundamental flaw in the logic of the argument. The option pricing theories, although they are based on arbitrage arguments, are consistent with the equilibrium ICAPM model of Merton (1973a). Under this theory expected rates of return are unrelated to idiosyncratic risk. Therefore in economies of well-diversified investors, option pricing theories do not imply that idiosyncratic volatility is a significant predictor of stock returns, provided that market risk is included as a regressor in the predictive model of returns.

3.4.4. *Entrepreneurial risk hypothesis*

Finally, consider the *entrepreneurial risk hypothesis*, which is attributable to Jaganathan and Wang (1996) and Heaton and Lucas (2000). The first authors recognize that a large proportion of wealth is in human capital. As a result, the risk of an equity portfolio is measured using two risk factors, namely, the stock market index return and the return on human capital.[11]

We formalize their theory in the single-factor Merton (1973a) ICAPM, writing the conditional expected excess return

$$\begin{aligned} E_{t-1}[R_{pt}] &= B\sigma_{pw,t-1} \\ &= b_1\sigma_{pm,t-1} + b_2\sigma_{pe,t-1}, \end{aligned} \tag{15}$$

where B is relative risk aversion of the representative investor. In the first line, the covariance of returns of portfolio p and the total wealth portfolio $\sigma_{pw,t-1}$ appear. Because total wealth is a weighted sum of stock market wealth and human capital, the risk can be written as the weighted sum of the covariances of portfolio p with the stock market return $\sigma_{pm,t-1}$ and with the growth rate of aggregate labor income $\sigma_{pe,t-1}$.[12] The coefficients in the second line are $b_1 = Bw_m$ and $b_2 = Bw_e$, where w_m and w_e are the proportions of aggregate wealth in the stock market and in human capital. These coefficients are interpreted alternatively as the market prices of the respective factor risks.

Heaton and Lucas (2000) study panel data and demonstrate that a large proportion of aggregate stock holdings is held by small business proprietors, while wage earners hold little stock. For this reason they argue that the second source of risk $\sigma_{pe,t-1}$ in Eq. (15) should be measured as the covariance of returns with the growth in proprietary business income, which we call entrepreneurial risk. Heaton and Lucas (2000) also study equity returns, and they demonstrate that differences in entrepreneurial risk are significant explanatory variables for the cross section of expected excess returns.

We study entrepreneurial risk for two reasons. First, we suspect that the time series of idiosyncratic volatility of small firms, namely $SMALL$, is positively correlated with the volatility of proprietary business income and therefore with entrepreneurial risk. We know from existing work that small

[11] There is a third factor in their model — the bond default risk premium — but this is not a risk factor. Instead, it is a proxy for the measurement error in unconditional betas.

[12] The return on human capital is not observable. Jaganathan and Wang (1996) and Heaton and Lucas (2000) finess this point by assuming that human capital is calculated in the constant growth model. It follows, then, that growth of aggregate income is equal to the return on aggregate human capital.

firms share common risk factors, and that the correlation of two small-firm returns is typically greater than the correlation of small- and big-firm returns. Given that most proprietorships are also small firms — albeit small *private* firms — it is expected that investment returns of proprietorships are more highly correlated with returns of small public corporations than with big-firm returns. This suggests that *SMALL* is likely to covary positively with the level of risk in ownership of entrepreneurial firms. In other words, the *entrepreneurial risk hypothesis* states that idiosyncratic volatility of small firms is a predictor of returns because it is a proxy for $\sigma_{pe,t-1}$ in Eq. (15).

We also study entrepreneurial risk because we wish to extend the work of Heaton and Lucas (2000). Whereas they describe the cross-section of expected excess returns, we study the conditional distributions of excess stock returns in time series. Our evidence in prior sections, like the evidence in the existing literature, demonstrates that a number of business-cycle instruments are predictors of returns, including bond market default spreads and term spreads. In the next section, we examine entrepreneurial risk as an explanatory variable for the times series of returns and ask if commonly used business cycle instruments are proxies for that risk.

4. Portfolio Returns and Entrepreneurial Risk

We generalize the empirical models in prior sections. Here, we study the projection of portfolio excess returns

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 SMALL_{t-1} + b_3 ENT_{p,t-1} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t. \tag{16}$$

As before, $MKT_{p,t-1}$ is the covariance of daily returns of portfolio p and the CRSP value-weighted index during month $t-1$, and as such is our empirical estimate of $\sigma_{pm,t-1}$, which appears in model (15). $SMALL_{t-1}$ is the average idiosyncratic risk of small stocks during month $t-1$, and similarly, the $X_{t-1}^{(j)}$ are instrumental variables measured in month $t-1$. The new variable here is $ENT_{p,t-1}$, which is an estimator of entrepreneurial risk $\sigma_{pe,t-1}$ and which we describe below. Equation (16) is a predictive model, projecting the month t portfolio return $R_{p,t}$ on variables observed in the preceding month $t-1$.

Under a strict interpretation of model (15), neither *SMALL* nor any of the instrumental variables have predictive power in Eq. (16), and their coefficients are zero. However, we have already seen evidence that *SMALL* and other variables including bond market term and default spreads are significant in projections that exclude entrepreneurial risk. Therefore, we focus our attention on changes in measures of significance of coefficients b_2 and c_j when ENT is added as a regressor. If the entrepreneurial risk hypothesis is valid, we expect b_2 is insignificantly different from zero when ENT appears as a regressor. Similarly, if the business cycle variables $X_{t-1}^{(j)}$ are proxies for entrepreneurial risk, we expect that the c_j are insignificantly different from zero after the inclusion of ENT.

4.1. *Income portfolios*

Because we do not observe either the value of or the return on human capital, it is necessary to construct an indirect estimator of entrepreneurial risk $ENT_{p,t-1}$. We have considered two alternative methods. The first relies on the arguments of Jaganathan and Wang (1996) and Heaton and Lucas (2000), who assume that the value of human capital follows the constant growth model of valuation. Using this model, they conclude that the growth rate of entrepreneurial business income, say $G_{e,t}$, is equivalent to and therefore can be used as a substitute for the return on human capital. In this case, $ENT_{p,t-1}$ is calculated as a covariance of monthly observations of the portfolio returns $R_{p,t}$ and $G_{e,t}$, perhaps using a GARCH model.

We follow a second method, which is developed by Breeden *et al.* (1989) and more recently by Lamont (2001). We construct a mimicking *income portfolio*, which in our case is a portfolio of stocks that best predicts future growth in proprietary business income. $ENT_{p,t-1}$ is calculated as the covariance of daily returns on portfolio p and the income portfolio during month $t-1$.

The second method has three advantages relative to the first. One is that a mimicking portfolio captures news in stock returns forecasting entrepreneurial income over long horizons. In our particular application, we alternatively consider income growth $G_{e,t,t+k}$ over one-, three- and five-year horizons. Thus, $ENT_{p,t-1}$ describes how the returns on portfolio p covary with news about income growth beyond the current month. Given that proprietary business income is strongly countercyclical to the business cycle, this particular characteristic of $ENT_{p,t-1}$ is important.

A second and related advantage is that we do not require that human capital follow the constant growth model. By regressing income over long intervals on monthly stock returns, we do not assume that the growth in proprietary business income is equal to the growth in human capital. Instead, we assume that the innovations in the income portfolio returns adequately capture the innovations in human capital.

A third advantage, which is emphasized by Breeden *et al.* (1989), is the opportunity to use data at higher frequencies than the monthly observations of income. We calculate $ENT_{p,t-1}$ using the same method as that for $MKT_{p,t-1}$ and $SMALL_{t-1}$, and this method uses monthly samples of daily returns.

Monthly aggregate non-farm proprietary income is from the National Income and Products Accounts (NIPA). We calculate the monthly growth rate in per-capita proprietary income, normalizing the income series by US total population.[13] We calculate the growth rate of income $G_{e,t,t+k}$ between months t and $t+k$, where $k = 1, 12, 36$ and 60. Descriptive statistics of $G_{e,t,t+1}$ appear in Panel A of Table 6.

We project growth rates using a model similar to that of Vassalou (2003),

$$G_{e,t,t+k} = a + \sum_{n=1}^{N} d_n B_t^{(n)} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t, \qquad (17)$$

where the $B_{t-1}^{(n)}$ are the excess returns of base assets and $X_{t-1}^{(j)}$ are the same business-cycle instruments used in our projections of stock returns. Our base assets are the six value-weighted stock portfolios of Fama and French (1993) constructed from the intersection of two size and three book-to-market portfolios.

Using the weights d_n estimated in (17), daily returns on the mimicking income portfolio are calculated

$$R_{e,s} = \sum_{n=1}^{N} d_n B_s^{(n)}. \qquad (18)$$

Finally, entrepreneurial risk for portfolio p during month t is calculated by summing over the T days in that month to obtain

$$ENT_{p,t} = \sum_{s=1}^{T} R_{e,s} R_{p,s}, \qquad (19)$$

where $p = \mathit{Small}$ or Big.

4.2. *Portfolio returns projections*

We use one-, three- and five-year horizons to estimate the mimicking portfolio weights d_n in Eq. (17). We then calculate R_e and ENT as in Eqs. (18) and (19). Finally, we report the results of the corresponding projections (16).

[13] Income and population data are in Table 2.6 of the Bureau of Economic Analysis website, http://www.bea.doc.gov/bea/dn/nipaweb/index.asp.

Tables 6–8 give results for the one-, three- and five-year horizons, respectively. In each table, Panel A holds descriptive statistics for monthly observations of the return on the mimicking portfolio R_e and of entrepreneurial risk ENT_p for both small and big firms. Panel B reports the mimicking portfolio weights, the $\overline{R}^2$ of the mimicking portfolio regression, and a Chi-square test of the hypothesis that all portfolio weights are jointly zero together with the p-value of the test. Panels C and D report the results of projections for small and big firms, respectively. In each of Panels C and D, rows (1)–(3) report cases where entrepreneurial risk ENT_p appears alone and cases where measures of volatility $SMALL$ and MKT also appear. Rows (4) and (5) include business-cycle instruments.

Standard calculations of t-tests and associated p-values for coefficients in Eq. (16) are not correct because they assume that the mimicking portfolio weights d_n are known and fixed, where in fact the weights are estimated values and are subject to error. Another way to put the problem is to recognize that because estimated coefficients d_n enter the regressor ENT in the predictive regression (16), the estimation problem is non-linear and standard OLS calculations of t-tests do not apply. For this reason, we calculate bootstrap p-values and report them within square brackets in Tables 6–8 immediately below the coefficients and Newey–West t-statistics, which are in parentheses. Appendix B contains a brief discussion of the calculations of the p-values.

Panel B of Tables 6–8 show marked differences in $\overline{R}^2$ of the mimicking portfolio regressions for the one-year case on the one hand, and the three- and five-year cases on the other. The $\overline{R}^2$ in the latter two cases are on the order of 11%, while in the one-year case it is less than 3%.

A comparison across the corresponding rows in Panel C of Tables 7 and 8 shows very little difference in the magnitudes of either the coefficients on ENT or their p-values between the three- and five-year horizons cases for small firms. A similar statement can be made for big firms comparing Panel D across tables. However, in comparing these tables to Table 6, we see more noticeable differences between the one-year case on the one hand and the three- and five-year horizon cases on the other. This is true for both small- and big-firm returns. Note in particular that the coefficients on ENT are generally smaller in Table 6 than in Tables 7 and 8. This evidence together with the comparison of $\overline{R}^2$ in Panel B suggests that a mimicking portfolio for entrepreneurial risk is better estimated using horizons for income growth longer than one year. Hence, we focus our comments on Table 7, which contains results for the three-year income horizon.

Table 6. Forecasts of size portfolios excess returns for the mimicking portfolio with one-year horizon.

Panel A: Income growth, mimicking portfolio returns and entrepreneurial risk

	Mean	Std Dev	Min	Max	Skew	Kurt	AR_1
G_e	0.5233	0.8052	−4.4857	5.5952	−0.0881	10.7229	0.4798
R_e	0.1454	0.5566	−1.8170	2.9247	0.1583	5.3509	0.0954
ENT_{Small}	0.0068	0.0178	−0.0584	0.3040	10.5272	175.2451	0.1768
ENT_{Big}	0.0063	0.0207	−0.0527	0.3969	14.6383	278.9982	0.1468

Panel B: One-year proprietary income mimicking portfolio

Small			*Big*			χ^2	*p-value*	$\overline{R}^2$
Low	*Medium*	*High*	*Low*	*Medium*	*High*			
−0.031	0.105	−0.026	−0.126	0.095	0.085	9.703	0.138	2.90%
(−0.278)	(0.431)	(−0.152)	(−0.866)	(0.569)	(0.825)			

Panel C: NYSE/AMEX/NASDAQ Small-firm excess returns (August 1963–December 2001)

	MKT	*SMALL*	*ENT*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	$\overline{R}^2$ (%)
(1)			0.426							1.61
			(3.216)							
			[0.029]							
(2)		0.478	0.437							2.35
		(1.888)	(3.096)							
		[0.084]	[0.034]							
(3)	−1.466	0.662	0.451							2.64
	(−1.608)	(2.262)	(3.144)							
	[0.175]	[0.032]	[0.032]							

Table 6. (*Continued*)

	MKT	*SMALL*	*ENT*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	$\overline{R}^2$ (%)
(4)		1.105	0.290	0.035	−4.086	−0.974	2.970	0.775	0.135	12.85
		(3.567)	(3.881)	(2.109)	(−1.637)	(−4.506)	(2.919)	(3.266)	(1.924)	
		[0.002]	[0.012]	[0.044]	[0.977]	[0.000]	[0.007]	[0.003]	[0.066]	
(5)	−1.719	1.419	0.291	0.042	−4.284	−1.015	2.986	0.747	0.105	13.14
	(−1.573)	(3.708)	(3.621)	(2.343)	(−1.664)	(−4.631)	(2.909)	(3.087)	(1.509)	
	[0.181]	[0.001]	[0.018]	[0.026]	[0.976]	[0.000]	[0.007]	[0.005]	[0.147]	

Panel D: NYSE/AMEX/NASDAQ Big-firm excess returns (August 1963–December 2001)

	MKT	*SMALL*	*ENT*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	$\overline{R}^2$ (%)
(1)			0.287							1.83
			(2.714)							
			[0.095]							
(2)		0.134	0.290							1.94
		(0.610)	(2.578)							
		[0.569]	[0.107]							
(3)	−1.335	0.340	0.307							2.62
	(−2.735)	(1.425)	(2.704)							
	[0.035]	[0.180]	[0.094]							
(4)		0.229	0.243	−0.001	−2.809	−0.569	2.649	0.812	−0.052	9.57
		(0.817)	(3.155)	(−0.102)	(−1.326)	(−3.731)	(3.669)	(4.688)	(−0.916)	
		[0.443]	[0.059]	[0.921]	[0.932]	[0.001]	[0.001]	[0.000]	[0.370]	

Table 6. (*Continued*)

	MKT	*SMALL*	*ENT*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	$\overline{R}^2$ (%)
(5)	−1.890	0.644	0.248	0.007	−3.209	−0.600	2.632	0.768	−0.085	10.63
	(−3.271)	(1.902)	(2.826)	(0.483)	(−1.443)	(−3.753)	(3.615)	(4.291)	(−1.761)	
	[0.011]	[0.078]	[0.081]	[0.640]	[0.916]	[0.001]	[0.002]	[0.000]	[0.092]	

Note: Panel A reports statistics for monthly growth in aggregate proprietary income from the NIPA G_e and monthly returns on a mimicking portfolio R_e. Panel A also reports statistics for monthly observations of entrepreneurial risk $ENT_{p,t}, p = \mathit{Small}$ and *Big*, which are covariances and are calculated as sums of products of daily returns of R_p and R_e as described in Sec. 2. The month $t-1$ return of the mimicking portfolio is $R_{e,t-1} = \sum_{n=1}^{N} d_n B_{t-1}^{(n)}$, where $B^{(n)}$ are monthly excess returns of six base assets that are intersections of two size and three book-to-market portfolios. Panel B reports the portfolio weights d_n, which are estimated by regressing monthly observations of one-year growth of proprietary income on predetermined variables in

$$G_{e,t,t+12} = a + \sum_{n=1}^{N} d_n B_{t-1}^{(n)} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t,$$

where the $X_{t-1}^{(j)}$ are business-cycle instruments defined in the text. Panel B also reports a χ^2 test of the hypothesis that the 6 portfolio weights are jointly zero. Panels C and D report estimates of slope coefficients and coefficients of determination from regressions of monthly returns

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 SMALL_{t-1} + b_3 ENT_{p,t-1} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t,$$

SMALL and MKT_p, $p = \mathit{Small}$ and *Big*, are calculated from sums of products of daily returns within months. *SMALL* is the value-weighted average of idiosyncratic variances of small firms, using the single index model. MKT_p is the covariance of R_p with the market index return. All (co)variance estimates are multiplied by 10^2. R_p, ENT_p and $MKT_p, p = \mathit{Small}$ and *Big* appear in Panels B and C, respectively. Newey–West t-statistics with six lags are in parentheses. Bootstrapped (two-sided) p-values using 10,800 replications are in brackets. See Appendix B for further details on the bootstrap experiment.

Table 7. Forecasts of size portfolios excess returns for the mimicking portfolio with three-year horizon.

Panel A: Mimicking portfolio returns and entrepreneurial risk

	Mean	Std Dev	Min	Max	Skew	Kurt	AR_1
R_e	0.0490	1.4415	−7.9717	5.2448	−0.3409	5.7960	0.0647
ENT_{Small}	0.0196	0.0492	−0.0589	0.8235	10.4745	160.8233	0.1664
ENT_{Big}	0.0152	0.0485	−0.0945	0.8838	12.8550	227.6741	0.1794

Panel B: Three-year proprietary income mimicking portfolio

	Small			*Big*			χ^2	*p-value*	$\overline{R}^2$
	Low	*Medium*	*High*	*Low*	*Medium*	*High*			
	0.210	0.140	−0.252	−0.473	0.702	−0.168	9.391	0.153	11.74%
	(1.012)	(0.360)	(−0.958)	(−1.780)	(2.665)	(−0.885)			

Panel C: NYSE/AMEX/NASDAQ Small-firm excess returns (August 1963–December 2001)

	MKT	*SMALL*	*ENT*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	$\overline{R}^2$ (%)
(1)			0.167							1.89
			(3.422)							
			[0.015]							
(2)		0.306	0.153							2.18
		(1.085)	(3.298)							
		[0.318]	[0.019]							
(3)	−1.390	0.476	0.157							2.44
	(−1.517)	(1.420)	(3.241)							
	[0.198]	[0.178]	[0.021]							
(4)		0.975	0.153	0.037	−4.103	−1.037	3.020	0.821	0.135	13.63
		(3.016)	(3.266)	(2.312)	(−1.651)	(−4.815)	(3.026)	(3.418)	(1.896)	
		[0.007]	[0.022]	[0.031]	[0.973]	[0.000]	[0.004]	[0.002]	[0.072]	

Table 7. (*Continued*)

	MKT	*SMALL*	*ENT*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	$\overline{R}^2$ (%)
(5)	−1.781	1.299	0.155	0.044	−4.307	−1.081	3.037	0.791	0.104	13.94
	(−1.712)	(3.275)	(3.014)	(2.623)	(−1.689)	(−5.024)	(3.028)	(3.238)	(1.475)	
	[0.140]	[0.003]	[0.029]	[0.015]	[0.971]	[0.000]	[0.004]	[0.004]	[0.161]	

Panel D: NYSE/AMEX/NASDAQ Big-firm excess returns (August 1963–December 2001)

	MKT	*SMALL*	*ENT*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	$\overline{R}^2$ (%)
(1)			0.126							1.94
			(3.068)							
			[0.045]							
(2)		0.038	0.125							1.95
		(0.167)	(3.085)							
		[0.880]	[0.045]							
(3)	−1.240	0.226	0.129							2.54
	(−2.425)	(0.851)	(3.115)							
	[0.058]	[0.418]	[0.044]							
(4)		0.168	0.134	0.001	−2.921	−0.632	2.686	0.847	−0.055	10.42
		(0.604)	(3.237)	(0.109)	(−1.374)	(−4.057)	(3.751)	(4.840)	(−0.969)	
		[0.572]	[0.039]	[0.918]	[0.922]	[0.000]	[0.001]	[0.000]	[0.357]	

Table 7. (*Continued*)

	MKT	$SMALL$	ENT	DP	RTB	TS	DS	CAY	R_m	$\overline{R}^2$ (%)
(5)	−1.881	0.581	0.136	0.010	−3.323	−0.664	2.669	0.804	−0.088	11.48
	(−3.470)	(1.694)	(2.897)	(0.695)	(−1.505)	(−4.159)	(3.734)	(4.443)	(−1.824)	
	[0.010]	[0.111]	[0.055]	[0.509]	[0.903]	[0.000]	[0.001]	[0.000]	[0.086]	

Note: Panel A reports statistics for monthly returns on a mimicking portfolio R_e. Panel A also reports statistics for monthly observations of entrepreneurial risk $ENT_{p,t}, p =$ *Small* and *Big*, which are covariances and are calculated as sums of products of daily returns of R_p and R_e as described in Sec. 2. The month $t-1$ return of the mimicking portfolio is $R_{e,t-1} = \sum_{n=1}^{N} d_n B_{t-1}^{(n)}$, where $B^{(n)}$ are monthly excess returns of six base assets that are intersections of two size and three book-to-market portfolios. Panel B reports the portfolio weights d_n, which are estimated by regressing monthly observations of three-year growth of proprietary income on predetermined variables in

$$G_{e,t,t+36} = a + \sum_{n=1}^{N} d_n B_{t-1}^{(n)} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t,$$

where the $X_{t-1}^{(j)}$ are business-cycle instruments defined in the text. Panel B also reports a χ^2 test of the hypothesis that the 6 portfolio weights are jointly zero. Panels C and D report estimates of slope coefficients and coefficients of determination from regressions of monthly returns

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 SMALL_{t-1} + b_3 ENT_{p,t-1} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t,$$

$SMALL$ and MKT_p, $p =$ *Small* and *Big*, are calculated from sums of products of daily returns within months as described in Sec. 1. $SMALL$ is the value-weighted average of idiosyncratic variances of small firms, using the single index model. MKT_p is the covariance of R_p with the market index return. All (co)variance estimates are multiplied by 10^2. R_p, ENT_p and $MKT_p, p =$ *Small* and *Big* appear in Panels B and C, respectively. Newey–West t-statistics with six lags are in parentheses. Bootstrapped (two-sided) p-values using 10,800 replications are in brackets. See Appendix B for further details on the bootstrap experiment.

Table 8. Forecasts of size portfolios excess returns for the mimicking portfolio with five-year horizon.

Panel A: Mimicking portfolio returns and entrepreneurial risk

	Mean	Std Dev	Min	Max	Skew	Kurt	AR_1
R_e	0.0270	1.9434	−10.5091	6.4894	−0.5158	5.5386	0.1069
ENT_{Small}	0.0218	0.0600	−0.0732	0.9008	8.4384	109.2619	0.1848
ENT_{Big}	0.0071	0.0405	−0.1958	0.5146	4.8318	60.4962	0.3186

Panel B: Five-year proprietary income mimicking portfolio

Small			*Big*			χ^2	*p-value*	$\overline{R}^2$
Low	*Medium*	*High*	*Low*	*Medium*	*High*			
0.254	0.447	−0.403	−0.614	0.818	−0.346	14.948	0.021	10.87%
(0.816)	(1.230)	(−1.319)	(−2.019)	(2.607)	(−1.333)			

Panel C: NYSE/AMEX/NASDAQ Small-firm excess returns (August 1963–December 2001)

	MKT	*SMALL*	*ENT*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	$\overline{R}^2$ (%)
(1)			0.131							1.73
			(4.187)							
			[0.004]							
(2)		0.288	0.118							1.98
		(0.964)	(3.637)							
		[0.367]	[0.007]							
(3)	−1.344	0.453	0.121							2.23
	(−1.435)	(1.296)	(3.614)							
	[0.214]	[0.218]	[0.008]							
(4)		0.960	0.132	0.038	−4.164	−1.046	3.053	0.839	0.135	13.75
		(2.874)	(3.300)	(2.395)	(−1.671)	(−4.842)	(3.083)	(3.470)	(1.893)	
		[0.010]	[0.013]	[0.022]	[0.972]	[0.000]	[0.004]	[0.002]	[0.071]	

Table 8. (*Continued*)

	MKT	*SMALL*	*ENT*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	$\overline{R}^2$ (%)
(5)	−1.794	1.287	0.133	0.045	−4.371	−1.090	3.071	0.810	0.104	14.07
	(−1.707)	(3.181)	(3.148)	(2.716)	(−1.709)	(−5.058)	(3.084)	(3.291)	(1.467)	
	[0.143]	[0.004]	[0.017]	[0.010]	[0.970]	[0.000]	[0.004]	[0.004]	[0.156]	

Panel D: NYSE/AMEX/NASDAQ Big-firm excess returns (August 1963–December 2001)

	MKT	*SMALL*	*ENT*	*DP*	*RTB*	*TS*	*DS*	*CAY*	R_m	$\overline{R}^2$ (%)
(1)			0.135							1.55
			(3.092)							
			[0.012]							
(2)		0.051	0.134							1.56
		(0.209)	(2.988)							
		[0.850]	[0.015]							
(3)	−1.133	0.224	0.135							2.06
	(−2.001)	(0.803)	(2.942)							
	[0.102]	[0.447]	[0.016]							
(4)		0.212	0.175	0.004	−3.090	−0.659	2.766	0.884	−0.060	10.80
		(0.731)	(5.220)	(0.280)	(−1.436)	(−3.974)	(3.828)	(4.956)	(−1.059)	
		[0.504]	[0.000]	[0.788]	[0.918]	[0.000]	[0.001]	[0.000]	[0.305]	

Table 8. (*Continued*)

	MKT	$SMALL$	ENT	DP	RTB	TS	DS	CAY	R_m	$\overline{R}^2$ (%)
(5)	−1.856	0.621	0.175	0.012	−3.489	−0.690	2.750	0.842	−0.093	11.83
	(−3.264)	(1.757)	(4.979)	(0.825)	(−1.564)	(−4.074)	(3.821)	(4.557)	(−1.916)	
	[0.013]	[0.101]	[0.000]	[0.421]	[0.900]	[0.000]	[0.001]	[0.000]	[0.062]	

Note: Panel A reports statistics for monthly returns on a mimicking portfolio R_e. Panel A also reports statistics for monthly observations of entrepreneurial risk $ENT_{p,t}$, $p =$ *Small* and *Big*, which are covariances and are calculated as sums of products of daily returns of R_p and R_e as described in Sec. 2. The month $t-1$ return of the mimicking portfolio is $R_{e,t-1} = \sum_{n=1}^{N} d_n B_{t-1}^{(n)}$, where $B^{(n)}$ are monthly excess returns of six base assets that are intersections of two size and three book-to-market portfolios. Panel B reports the portfolio weights d_n, which are estimated by regressing monthly observations of five-year growth of proprietary income on predetermined variables in

$$G_{e,t,t+60} = a + \sum_{n=1}^{N} d_n B_{t-1}^{(n)} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t,$$

where the $X_{t-1}^{(j)}$ are business-cycle instruments defined in the text. Panel B also reports a χ^2 test of the hypothesis that the 6 portfolio weights are jointly zero. Panels C and D report estimates of slope coefficients and coefficients of determination from regressions of monthly returns

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 SMALL_{t-1} + b_3 ENT_{p,t-1} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t,$$

$SMALL$ and MKT_p, $p =$ *Small* and *Big*, are calculated from sums of products of daily returns within months. $SMALL$ is the value-weighted average of idiosyncratic variances of small firms, using the single index model. MKT_p is the covariance of R_p with the market index return. All (co)variance estimates are multiplied by 10^2. R_p, ENT_p and MKT_p, $p =$ *Small* and *Big* appear in Panels B and C, respectively. Newey–West t-statistics with six lags are in parentheses. Bootstrapped (two-sided) p-values using 10,800 replications are in brackets. See Appendix B for further details on the bootstrap experiment.

Panel A of Table 7 reports the sample average and standard deviation of the excess return on the mimicking portfolio. Because these values are calculated using the portfolio weights in Panel B, which do not sum to one, we normalize the estimates dividing each by the sum 0.1595. This provides estimates of expected return and standard deviation that can be compared to those for the market index and size portfolios.

The average market-index excess return in Table 1 is more than 0.50% per month, and the small-firm excess return is more than 0.90% per month. Sharpe ratios of the market and small-firm portfolios — which are expected excess returns divided by standard deviation — are 0.123 and 0.156 respectively. In comparison, the average return on the mimicking portfolio using normalized weights is 0.31% per month, and the portfolio has a Sharpe ratio of 0.034, much less than those of the benchmarks.

Next, examine the coefficients of *ENT* in Panels C and D of Table 7, along with the t-statistics and the corresponding bootstrap p-values. Several points are relevant here. First, each coefficient on *ENT* in each row is positive and is significantly different from zero at the 1% level using the robust standard errors, and more importantly, each is significant at the 5% level using the more reliable bootstrap p-values, with the exception of row (5) of Panel D, where the bootstrap p-value is 5.5%. Second, the magnitudes of the coefficients on *ENT* are little different across rows in Panel C, and similarly across rows in Panel D.

Second, based on these facts we conclude that variation in entrepreneurial risk through time is a significant determinant of expected stock returns for both big and small firms. The positive coefficients demonstrate that expected returns increase directly with entrepreneurial risk, which is measured by the covariance of returns with mimicking portfolio returns. The results also suggest that *ENT* is not a proxy for the business-cycle instruments commonly used to predict stock returns; if it were a proxy, its coefficient would differ across the rows and it likely would be insignificant in rows (4) and (5) of Panels C and D.

Finally, Table 7 supports the entrepreneurial risk hypothesis. It shows that *SMALL* predicts stock returns because it is a proxy for entrepreneurial risk. In the presence of *ENT* as a regressor, *SMALL* is not a significant predictor in the projections reported in rows (1)–(3), where business-cycle instruments are excluded as regressors. Furthermore, the addition of entrepreneurial risk *ENT* as a regressor leads to a reduction in the size of the coefficient on *SMALL*. To see this, compare the coefficients in Panels A and B of Table 2 — where small- and big-firm returns are projected on *MKT* and *SMALL* only — to the results in Table 7. As a specific example, compare row (1) of

Panel A of Table 2 to row (2) of Panel C of Table 7. In the second column of Panel A of Table 2 the coefficient on *SMALL* is from a projection of small-firm returns on *SMALL* alone, while in row (2) of Panel C of Table 7 the same returns are projected on *SMALL* and *ENT* together. In Table 2, *SMALL* is 0.738 with a t-statistic of 2.839, while in Table 7 it is 0.306 with a t-statistic of 1.085 and a p-value of 0.318. This and other similar comparisons demonstrate that *SMALL* predicts returns because it is correlated with entrepreneurial risk, and not because idiosyncratic risk is priced.

Table 9 reports regressions analogous to those described by Table 7, except that size-decile portfolio returns are used. As in Table 7, *ENT* is calculated using three-year growth rates in proprietary business income, and in a manner similar to Tables 3 and 5 only the coefficients on *SMALL* and *ENT* are reported in Table 9. Four separate cases appear in the four panels, and these differ in the regressors that are used. In Panel A of Table 9 only *SMALL* and *ENT* appear as regressors, in Panel D *MKT* and business-cycle instruments appear as well, while in Panels B and C *MKT* and the collection of instruments are excluded separately.

Table 9 supports the conclusions drawn from Table 7. In all reported results, *ENT* is a positive predictor of returns, demonstrating that expected returns covary directly with entrepreneurial risk. Furthermore, *ENT* differs little across the panels of Table 9, suggesting that it is uncorrelated with the other regressors and contains independent information about the decile-portfolio returns. In Panels A and B, where business-cycle instruments are absent, *ENT* is a significant predictor of returns at the 5% level for stocks in deciles 4–9, and at the 10% level for deciles 2, 3, and 10 (big), and is insignificant at conventional levels for portfolio 1 (small). Very similar remarks can be made about Panels C and D, where business-cycle variables are included, although the bootstrap p-values are marginally higher. Given that the coefficient estimates generally are larger in Panels C and D than in Panels A and B, the larger p-values are the consequence of increased noise in the bootstrap experiment (apparently the result of the added regressors). From these results, we conclude that *ENT* is a positive and significant predictor of returns for all but the very smallest firms.

Table 9 also supports our contention that the significance of *SMALL* in predictive regressions is explained by the entrepreneurial risk hypothesis. Consider again Panels A and B, where business cycle instruments are absent. *SMALL* is not significant at the 5% level for any of the decile portfolios, although it is significant at the 10% level for portfolios 1 and 2. Compare these results to Panel A of Table 3, where *ENT* does not appear as a

Table 9. Forecasts of size decile portfolios excess returns for the mimicking portfolio with three-year horizon.

		Small	2	3	4	5	6	7	8	9	Big
Panel A: ($b_1 = 0, c_j = 0$)											
(1)	*SMALL*	0.482	0.492	0.301	0.158	0.149	0.126	0.210	0.271	0.187	−0.021
		(1.483)	(1.791)	(1.274)	(0.683)	(0.638)	(0.598)	(0.890)	(1.203)	(0.981)	(−0.089)
		[0.174]	[0.099]	[0.240]	[0.533]	[0.549]	[0.575]	[0.404]	[0.266]	[0.370]	[0.937]
(2)	*ENT*	0.068	0.126	0.133	0.156	0.149	0.146	0.149	0.129	0.128	0.114
		(1.364)	(2.341)	(2.544)	(3.295)	(3.022)	(3.123)	(3.424)	(3.204)	(3.167)	(2.876)
		[0.289]	[0.097]	[0.079]	[0.032]	[0.048]	[0.041]	[0.027]	[0.036]	[0.043]	[0.054]
Panel B: ($c_j = 0$)											
(1)	*SMALL*	0.749	0.679	0.443	0.290	0.245	0.208	0.339	0.403	0.353	0.211
		(1.970)	(2.064)	(1.564)	(1.018)	(0.854)	(0.803)	(1.211)	(1.492)	(1.576)	(0.768)
		[0.066]	[0.051]	[0.137]	[0.338]	[0.410]	[0.443]	[0.249]	[0.157]	[0.132]	[0.464]
(2)	*ENT*	0.074	0.130	0.136	0.158	0.151	0.147	0.152	0.132	0.132	0.119
		(1.415)	(2.384)	(2.575)	(3.331)	(3.073)	(3.172)	(3.446)	(3.259)	(3.183)	(2.901)
		[0.274]	[0.093]	[0.076]	[0.031]	[0.045]	[0.039]	[0.027]	[0.034]	[0.042]	[0.053]
Panel C: ($b_1 = 0$)											
(1)	*SMALL*	1.208	1.085	0.962	0.747	0.673	0.674	0.616	0.686	0.503	−0.008
		(3.068)	(3.265)	(3.321)	(2.704)	(2.485)	(2.651)	(2.263)	(2.337)	(1.952)	(−0.028)
		[0.007]	[0.004]	[0.003]	[0.014]	[0.027]	[0.018]	[0.038]	[0.036]	[0.076]	[0.980]
(2)	*ENT*	0.070	0.127	0.136	0.159	0.154	0.150	0.155	0.134	0.135	0.125
		(1.482)	(2.554)	(2.638)	(3.399)	(3.103)	(3.119)	(3.582)	(3.277)	(3.270)	(3.064)
		[0.258]	[0.075]	[0.074]	[0.032]	[0.043]	[0.043]	[0.024]	[0.033]	[0.038]	[0.045]

Table 9. (*Continued*)

		Small	2	3	4	5	6	7	8	9	Big
Panel D											
(1)	*SMALL*	1.536	1.389	1.265	1.046	0.930	0.944	0.945	1.090	0.928	0.461
		(3.263)	(3.308)	(3.599)	(3.090)	(2.765)	(2.946)	(2.742)	(3.034)	(2.872)	(1.252)
		[0.003]	[0.003]	[0.002]	[0.006]	[0.010]	[0.008]	[0.013]	[0.005]	[0.008]	[0.242]
(2)	*ENT*	0.071	0.128	0.137	0.160	0.155	0.151	0.156	0.135	0.136	0.127
		(1.406)	(2.401)	(2.472)	(3.162)	(2.925)	(2.918)	(3.273)	(2.933)	(2.896)	(2.704)
		[0.282]	[0.089]	[0.090]	[0.041]	[0.052]	[0.053]	[0.033]	[0.049]	[0.056]	[0.066]

Note: Estimates of the slope coefficient b_2 and b_3 of

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 SMALL_{t-1} + b_3 ENT_{p,t-1} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t,$$

are shown in row (1) and (2), respectively in each Panels A, B, C and D. R_p for $p = 1, \ldots, 10$ are monthly excess returns of value-weighted portfolios of firms grouped into deciles according to market capitalization. $SMALL$ and MKT_p, $p = 1, \ldots, 10$, are calculated from sums of products of daily returns within months as described in Sec. 1. $SMALL$ is the value-weighted average of idiosyncratic variances of small firms, using the single index model. Small firms have market capitalization less than the median market capitalization of NYSE issues in our data in a month. MKT_p is the covariance of R_p with the market index excess return. $ENT_{p,t}$, $p = 1, \ldots, 10$ are covariances and are calculated as sums of products of daily excess returns R_p and R_e, where $R_e = \sum_{n=1}^{N} d_n B^{(n)}$ is the excess return on a mimicking portfolio. The $B^{(n)}$ are excess returns of six base assets that are intersections of two size and three book-to-market portfolios. The portfolio weights d_n are estimated by regressing monthly observations of three-year growth of aggregate proprietary income $G_{e,t,t+36}$ on predetermined variables in

$$G_{e,t,t+k} = a + \sum_{n=1}^{N} d_n B_{t-1}^{(n)} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t.$$

The $X_{t-1}^{(j)}$ are business-cycle instruments defined in the text. All (co)variance estimates are multiplied by 10^2. Panels A–D differ according to whether the coefficients b_1 and c_j are alteratively set to zero or estimated. Newey–West t-statistics with six lags are in parentheses. Bootstrapped (two-sided) p-value using 10,800 replications are in brackets. See Appendix B for further details on the bootstrap experiment.

regressor. *SMALL* is a significant predictor of decile portfolio returns in Table 3, and gets a much larger coefficient for most decile portfolios. In other words, the addition of *ENT* to the regression cuts the magnitude of the coefficient on *SMALL* and reduces its significance.

As in Table 7, there is evidence in Table 9 that *SMALL* contains business-cycle information that is orthogonal to the portfolio returns. Compare the coefficients on *SMALL* in Panels A and B of Table 9 to the corresponding values in Panels C and D, where business cycle instruments appear. *SMALL* is not a significant predictor of returns in Panels A and B. In Panels C and D, the magnitude of *SMALL* is considerably larger and it is significantly different from zero at the 5% level using the bootstrap p-values.

Finally, compare the magnitudes of the coefficients across the columns of Table 9. The coefficients on *ENT* are positive and differ little across size-decile portfolios with the exception that for portfolio 1 the coefficient is about half those for portfolios 2–10. This is evidence that expected returns increase with entrepreneurial risk for companies of all sizes.

5. Summary and Final Comments

We first characterize the predictive power of the average idiosyncratic volatility of small public firms, *SMALL*, for stock returns. We find that the idiosyncratic volatilities of small firms, but not those of big firms, predict future portfolio returns. Importantly, we demonstrate that *SMALL* is a significant predictor of both small- and big-firm returns.

We consider four hypotheses — imperfect diversification, illiquidity, option, and entrepreneurial risk hypotheses — to explain the predictive power of *SMALL*. Our evidence offers no support for the first three hypothesis, but it supports the fourth. The entrepreneurial risk hypothesis, which we attribute to Jaganathan and Wang (1996) and Heaton and Lucas (2000), says that a critical risk factor in the stock market is the return on the human capital of small-business proprietors. We show that *SMALL* predicts returns because it is a proxy for entrepreneurial risk.

We calculate portfolio returns that mimic stock-market news for proprietors' future income. We calculate the covariance of portfolio returns with the return on the mimicking portfolio, namely *ENT*, and we use this to represent entrepreneurial risk. We project stock portfolio returns on *SMALL*, *ENT*, and a collection of instrumental variables representing the business cycle and variations in stock market liquidity. Evidence that *SMALL* is a proxy for *ENT* appears in bivariate regressions where *ENT* is a significant predictor of portfolio returns and *SMALL* is insignificant. More importantly, *ENT* is a significant predictor in regressions where it appears jointly with commonly used business-cycle instruments. That is, there is evidence that expected stock returns are increasing functions of covariances of returns with proprietors' human capital.

Whereas Jaganathan and Wang (1996) and Heaton and Lucas (2000) characterize the cross section of expected returns, our evidence characterizes the time series of expected returns. As a result of our work, we believe that the conditional expected returns of stocks vary directly with the level of risk in the economy faced by proprietors of private businesses. We believe that this is true of both large and small firms.

Broadly construed, our hypotheses are not mutually exclusive. For example, entrepreneurial risk exists in part because investors in public corporations who are also proprietors are often imperfectly diversified; they hold significant positions in their private businesses. Also, private businesses are highly illiquid assets when compared to stock of public corporations. Therefore, we might expect the distributions of returns of proprietorships and stocks to reflect both illiquidity and imperfect diversification in addition to entrepreneurial risk. However, we interpret our hypotheses narrowly and within the context in which they are proposed.

Merton (1987) argues that investors hold imperfectly diversified portfolios of stocks when information is limited, and in so doing they require compensation in their stock portfolios for the nonsystematic risk they face. In a similar vein, Amihud and Mendelson (1986) argue that the cross section of stock returns reflects differences in stock market liquidity. Neither of these theories is a statement about the implications of small businesses on stock returns. Therefore, we consider the *entrepreneurial risk* hypothesis as separate from them. Our analysis and tests demonstrate the economic significance of our measure of entrepreneurial risk *ENT* as a predictor of returns. We leave for future work to study the relative importance of the competing theories and to develop new theories for stock returns generally.

One final comment concerns investors who do not naturally face entrepreneurial risk. These investors can take advantage of the variations in expected returns by tilting their portfolios toward stocks when levels of entrepreneurial risk are large. Given that expected returns increase with entrepreneurial risk and given that these investors do not suffer this risk (unlike small business proprietors), a carefully executed strategy might hold total risk constant and yet obtain higher average returns than a naive strategy. This also is a topic for future research.

Acknowledgment

We thank Jose Miguel Gaspar, Eric Ghysels, Amit Goyal, Mark Ready, Maria Vassalou, Samuel Thompson and João Pedro Pereira for their helpful comments. We have benefited from the comments of participants at meetings of the 2004 European Finance Association, 2004 Portuguese Finance Network, and 2005 American Finance Association.

Appendix A. Additional Robustness Checks

To examine the robustness of the results of Sec. 3, we replicate the work using a number of alternative models and data subperiods. Using monthly data we repeatedly replicate the regressions reported in row (3) of panels A (for small firms) and B (for big firms) in Table 4. In all these regressions, all Table 4 variables except for the illiquidity variables are included as regressors.

Table A.1 reports the estimates for MKT and $SMALL$, along with the adjusted R-squares and Newey–West t-statistics; other estimated coefficients are not reported.

A.1. *Models of returns*

Rows (1) and 2 of Table A.1 report results using Eq. (5) to calculate idiosyncratic volatility according to the Fama and French (1992) three-factor model and an industry model. In the first case, two additional regressors appear in the predictive regression of the portfolio returns, but these are not shown in Table A.1. They are the covariances of the portfolio return with the factors HML and SMB. The value reported as MKT in row (1) is the coefficient for the covariance of the portfolio return with the market index, while the other coefficients are not reported. In the case of the industry model, only the covariance with the market index is included as a regressor (along with the controls and $SMALL$), and its coefficient is reported as MKT.

The results are broadly consistent with our previous findings. $SMALL$ is a significant and positive predictor of excess small- and big-firm returns.

A.2. *Volatility measures*

French *et al.* (FSS) (1987) suggest the estimator for variance:

$$\hat{\sigma}^2_{k,t} = \sum_{s=1}^{T} R^2_{k,s} + 2\sum_{s=2}^{T} R_{k,s}R_{k,s-1}, \tag{A.1}$$

Table A.1. Forecasts of excess returns.

NYSE/AMEX/NASDAQ		Small-Firm Excess Return			Big-Firm Excess Return		
		MKT	*SMALL*	$\overline{R}^2$ (%)	*MKT*	*SMALL*	$\overline{R}^2$ (%)
Panel A: Volatility models (August 1963–December 2001)							
(1)	Fama–French three factors	−1.155	2.284	11.14	−3.468	1.449	5.96
		(−0.409)	(4.243)		(−2.502)	(3.299)	
(2)	Industry factors	−2.332	2.061	11.25	−2.232	1.201	5.98
		(−1.855)	(4.471)		(−4.008)	(3.333)	
Panel B: Volatility measures (August 1963–December 2001)							
(3)	FSS correction	−1.293	1.972	10.06	−1.707	1.027	5.20
		(−1.633)	(4.012)		(−4.527)	(2.720)	
(4)	Linearly detrended	−1.054	1.249	8.16	−1.632	0.446	3.90
		(−0.840)	(2.685)		(−2.529)	(1.202)	
(5)	MIDAS	−3.554	1.397	17.27	−0.669	0.735	8.52
		(−0.927)	(5.992)		(−0.360)	(4.470)	
Panel C: Subsamples							
(6)	Sample 1963–1981	1.270	−0.384	12.65	−0.751	−0.133	11.20
		(0.199)	(−0.290)		(−0.218)	(−0.156)	
(7)	Sample 1982–2001	−2.132	1.826	11.57	−2.287	0.862	4.41
		(−1.787)	(3.118)		(−3.055)	(1.776)	
(8)	Sample 1963–1999	−3.135	2.241	11.32	−2.885	1.532	7.20
		(−3.197)	(4.726)		(−4.884)	(3.963)	
(9)	Sample 1963–2001	−0.262	1.806	11.24	−0.619	0.927	5.40
	(No Oct 1987 Crash)	(−0.085)	(4.099)		(−0.326)	(2.564)	

Table A.1. (*Continued*)

NYSE/AMEX/NASDAQ		Small-Firm Excess Return			Big-Firm Excess Return		
		MKT	*SMALL*	$\overline{R}^2$ (%)	*MKT*	*SMALL*	$\overline{R}^2$ (%)
Panel D: Other portfolios							
(10)	Value-line futures	−2.180	1.558	6.59			
		(−3.508)	(2.935)				
(11)	S&P 500				−2.204	0.978	5.43
					(−3.849)	(2.924)	
Panel E: Other risk factors (August 1963–December 1999)							
(12)	PS liquidity risk factor	−3.204	2.066	10.93	−2.720	1.236	6.25
		(−3.284)	(4.730)		(−4.439)	(3.121)	
(13)	VX default likelihood factor	−3.448	2.386	12.87	−3.112	1.596	7.91
		(−3.459)	(4.615)		(−4.790)	(3.798)	
Panel F: Liquidity measures (August 1963–December 2001)							
(14)	Turnover	−2.385	1.863	11.90	−2.452	1.018	6.93
		(−1.904)	(4.225)		(−3.623)	(2.898)	
(15)	Dollar volume	−1.967	1.930	12.58	−2.365	1.058	8.50
		(−1.374)	(4.563)		(−3.220)	(3.067)	

Table A.1. (*Continued*)

NYSE/AMEX/NASDAQ		Small-Firm Excess Return			Big-Firm Excess Return		
		MKT	*SMALL*	$\overline{R}^2$ (%)	*MKT*	*SMALL*	$\overline{R}^2$ (%)
Panel G: Seasonality (August 1963–December 2001)							
(16)	January dummy	−1.902	1.810	12.89	−2.319	1.099	5.80
		(−1.553)	(4.065)		(−3.792)	(3.155)	
(17)	May–November dummy	−2.140	1.933	11.53	−2.329	1.107	5.83
		(−1.886)	(4.451)		(−3.894)	(3.234)	

Note: Estimates of slope coefficients of

$$R_{p,t} = a + b_1 MKT_{p,t-1} + b_2 SMALL_{t-1} + \sum_{j=1}^{J} c_j X_{t-1}^{(j)} + \epsilon_t,$$

are shown where *SMALL* is the average of idiosyncratic volatilities of small-firm returns. Small firms have market capitalization less than the median market capitalization of NYSE issues in our data in a month. MKT_p is the covariance of the market excess return with the size portfolio excess return (covariances of the SMB and HML factors with the size portfolio excess returns are also included as regressors in the case of the Fama–French three-factor model). All volatility series are value-weighted monthly variances. $X_{t-1}^{(j)}$ denotes the business cycle variables as defined in text (coefficients not shown in the table). Panel A reports results using idiosyncratic volatility estimates from Fama–French three-factor model of returns, industry model of returns according to the 48 industry classification scheme in Fama and French (1997), and total volatility estimates obtained from daily data. Panel B reports results using alternative measures of volatility: variance with French *et al.* (1987) (FSS) autocorrelation correction, linearly detrended variance, and quarterly variance. MIDAS is the estimator of systematic volatility in Ghysels *et al.* (2005). Panel C reports results for subsamples and sample excluding the 1987 October crash. Panel D reports results using the maxi value line futures returns (March 1982–December 1999) and the S&P 500 excess return as dependent variables. Panel E reports results using as additional control variables, respectively, the Pastor and Stambaugh (2003) liquidity measure (August 1963–December 1999) and the Vassalou and Xing (2004) default risk (survival rate) measure (January 1971–December 1999). Panel F reports results including alternatively average (value-weighted) of turnover and dollar volume of small-firms. Panel G reports results including January dummy and May–November dummy. Newey–West t-statistics with six lags are in parentheses.

while Scholes and Williams (1977) suggest the estimator for covariance:

$$\hat{\sigma}_{kj,t} = \sum_{s=1}^{T} R_{k,s}R_{j,s} + \sum_{s=2}^{T} R_{k,s-1}R_{j,s} + R_{k,s}R_{j,s-1}. \tag{A.2}$$

The additional terms adjust for biases that result from autocorrelation and cross autocorrelations of daily returns.[14] Row (3) of Panel B of Table A.1 reports results using the market model Eq. (3) for idiosyncratic volatility, but using Eqs. (A.1) and (A.2) instead of Eqs. (7) and (8) to calculate variances and covariances of excess returns. In row (3) *SMALL* remains a positive and significant predictor of small- and big-firm returns.

Row (4) of Panel B of Table A.1 reports results using a linearly detrended time series of *SMALL*, which represents cyclical variations in *SMALL* but excludes the secular increase found by Campbell *et al.* (2001). *SMALL* is a positive and significant predictor of small-firm returns. *SMALL* is also a positive predictor of big-firm returns but is not statistically significant.

Row (5) of Table A.1 considers the MIDAS estimator for systematic volatility. Ghysels *et al.* (2005) find a positive and significant relation between systematic risk and monthly stock market returns using the MIDAS estimator. We use a MIDAS estimate of systematic risk, which is the covariance of small-firm and market excess returns, using a one-year window of returns. In this case, *SMALL* is a significant predictor of monthly small- and big-firm returns when included jointly with the MIDAS systematic volatility. Systematic volatility *MKT* is negative but insignificant.

A.3. *Subsamples*

Rows (6)–(9) of Table A.1 report results for several subperiods. Rows (6) and (7) break the full period into two equal halves, while row (8) excludes the years 2000 and 2001, which is a period of exceptionally high idiosyncratic volatility. Row (9) drops the October 1987 crash observation.

The results excluding the exceptional years 2000 and 2001 (row 8) and those excluding the October 1987 crash (row 9) are consistent with our

[14] The sum of cross-products in Eq. (A.1) serves two purposes. Returns of individual issues are serially correlated as the result of the bid-ask spread. This is especially true for low-priced and illiquid issues. Returns of the market portfolio and other indexes are serially correlated as the result of nonsynchronous trading. An estimator that is appropriate when there is zero correlation — i.e., the sum of squared returns in Eq. (7) — is biased when serial correlation is not zero. The sum of cross-products adjusts for this bias.

previous findings. *SMALL* is a significant and positive predictor of excess small- and big-firm returns.

When we divide the full period into two equal halves, we see some differences. *SMALL* is a significant predictor of small-firm returns only in the second half (1982–2001). However, in results that are not shown in the tables, *SMALL* is a positive and significant predictor of small-firm monthly returns in the first half (1962–1981) when it is a joint regressor with *MKT*, but without business-cycle controls. This is consistent with the results in Table 4. *SMALL* is a positive predictor of big-firm returns in the second half (1982–2001), but insignificant.

A.4. *Other portfolios*

The Q_{12} statistics in Table 1 show evidence that small-firm returns are highly predictable as a consequence of infrequent trading, whereas big-firm returns are not. We address this concern using returns of Value Line futures as suggested by Boudoukh *et al.* (1994), which do not suffer from infrequent trading. Returns during the period March 1982 through December 1999 are constructed using the contract closest to expiration, but not in the expiration month. We also consider the returns of the S&P 500 Index.

Rows (10) and (11) of Panel D of Table A.1 report the results. The results are consistent with our previous findings. *SMALL* is a positive and significant predictor of the Value Line futures returns and the S&P 500 Index returns.

A.5. *Other risk factors*

If the model of returns used to estimate idiosyncratic volatility is misspecificed, our estimates of idiosyncratic risk can be biased. Other risk factors have been suggested in the asset pricing literature. Two promenient examples are the liquidity risk factor of Pastor and Stambaugh (2003) and the default risk factor of Vassalou and Xing (2004). In principle, we could reestimate idiosyncratic volatility using this additional factor, but this is not possible because these factors are available only at a lower frequency (monthly). Alternatively, we run the predictive regression using as an additional control the monthly series of the liquidity risk factor and default risk factor.[15] We see that *SMALL* is a positive and significant predictor of small- and big-firm returns in both cases.

[15] We thank Lubos Pastor and Maria Vassalou for providing the data.

A.6. *Liquidity measures*

Rows (14) and (15) of Panel F report results using measures of liquidity, namely volume and dollar turnover as regressors in addition to the business cycle instruments. We see that *SMALL* is a positive and significant predictor of small- and big-firm returns in both cases.

A.7. *Seasonality*

Finally, rows (16) and (17) of Panel G report results where seasonal dummies appear as regressors. We consider a January dummy, recognizing the *small-firm-in-January* effect in row 16, and a May–November dummy recognizing that average market returns are higher in the winter and spring months than during the summer and fall months. Again, *SMALL* is a positive and significant predictor of small- and big-firm returns.

Appendix B. Bootstrap p-Values

We use bootstrap to estimate p-values for tests of hypotheses that coefficients in the predictive regression (16) are zero. To simplify discussion, we write the regression in matrix form

$$R_p = b_p ENT_p + c_p X + \epsilon_p, \tag{B.1}$$

where we include in X the variables *SMALL* and *MKT* and we write the coefficient b_3 as b_p here. The outline of our calculations of simulated values of b_p follows. The same steps are followed for the other coefficients.

Here is a brief outline of our steps:

(1) Repeatedly create simulated data sets of the same size as our dataset, and estimate $\hat{b}_p$ under the condition that $b_p = 0$. Do this, say, M times.
(2) Take the estimate $\hat{b}^*_p$ that we obtain from the data and that we report in the paper, and count the relative frequency of simulated values

$$|\hat{b}_p| > |\hat{b}^*_p|. \tag{B.2}$$

(3) Suppose this is the case in $P\%$ of the M simulations. Then $P\%$ is the two-sided bootstrap p-value and is an estimate of the probability of obtaining a coefficient estimate greater in magnitude than the observed value under the condition that the true value is zero.

We face a significant and important issue in running the simulations, namely the observations of the data are not independent. We know in particular that tests for zero autocorrelation in the control variables and our measures of volatility are rejected. As a result, we suspect that the OLS errors in the predictive regressions are not independent. If our dataset was a collection of independent observations, we would create M simulated datasets by randomly drawing with replacement from our dataset T times in each of the M simulations. Each draw would be one month t from the data. However, we must account for the persistence in the data when we run the simulation.

Hall and Horowitz (1996) describe a method of simulation for estimating standard errors of t statistics and GMM hypothesis tests. Following their work, we draw non-overlapping blocks of data randomly from our data and with replacement. Lengths of blocks of data are chosen to balance the desire to capture dependence in the data with the desire to have a large sample of blocks.

To construct blocks, observations in the population are indexed by $t = 1$ for August 1963 to $t = 461$ for December 2001. An observation in the population, say, for month t is

$$Z_t \equiv \{B_t, G_{e,t,t+36}, X_{t-1}, \epsilon_{pt}\}, \tag{B.3}$$

where $G_{e,t,t+36}$ is income growth from month t to month $t + 36$; B_t is the vector of returns of the base assets (used in construction of the mimicking portfolio); and X_{t-1} is the vector of controls, which includes a constant, the business cycle controls, *SMALL* and *MKT* as discussed previously. Also, ϵ_{pt} is the residual for month t from the return regression (16); the portfolio return R_p does not appear, but it will be constructed as described below. Note that the last observation of income growth, which is for $t = 461$, runs from December 2001 to December 2004. Also note that the earliest observation $t = 1$ includes the controls for July 1963.

We draw samples using blocks of observations either 35 or 36 months in length. This number of months is roughly the length of one-half of a business cycle and also is the length of the period over which we measure income growth $G_{e,t,t+36}$ for the income regression. Given that we have 461 observations, we make 6 blocks of data 36 months in length, and 7 blocks of 35 months each. For different simulation draws, blocks begin and end at different months and in some cases will begin in months 426 through 461. The result is that we append data to the end of the sample using the data at the beginning of the sample to create blocks of 35 or 36 months in length. For this reason, we extend the month index to $t = 497$, and then we make observation t equivalent to observation $t - 461$ for indexes $t = 462$–497.

A draw of a sample from the population is done using non-overlapping blocks of data. This is accomplished by creating three indexes m, i and j, as follows:

Draw (m)	Cycle (i)	First Observation in Block	
		Blocks $j = 1$–6	Blocks $j = 7$–13
1	1	$\{1, 37, 73, 109, 145, 181\}$	$\{217, 252, 287, 322, 357, 392, 427\}$
2	2	$\{2, 38, 74, 110, 146, 182\}$	$\{218, 253, 288, 323, 358, 393, 428\}$
...	...	...	...
36	36	$\{36, 72, 108, 144, 180, 216\}$	$\{252, 287, 322, 357, 392, 427, 462\}$
37	1	$\{1, 37, 73, 109, 145, 181\}$	$\{217, 252, 287, 322, 357, 392, 427\}$
...	...	...	...
m	$i = \text{mod}(m, 37)$	$i + (j - 1) \times 36$	$i + 6 \times 36 + (j - 7) \times 35$

Here, m is a index of the simulation draw. Let the maximum value be a multiple of 36, for example, let $m = 1, \ldots, 300 \times 36$, so there are 10,800 repeated estimations of coefficients. The index i is the cycle index, which picks the first observation of the first block. Note that i cycles from 1 through 36. Given that the number of draws is 300×36, there are 300 draws where we begin the first block at $t = 1$, 300 draws beginning at $t = 2$, and so on. Finally, index j identifies the block; $j = 1, \ldots, 13$.

In the body of the table are examples of the index t for the first observation in a block. Each of the first 6 blocks are 36 months in length, while the last 7 blocks are 35 months in length. The formulas for the first observation in a block are shown in the last row of the table.

For each draw m, blocks are sampled randomly, with replacement, and according to probabilities that are equal to the relative number of months in the block. Blocks are drawn until there are at least 461 observations. If on the final draw the bootstrap sample contains $n > 461$ observations, then the last $n - 461$ months of the last block are deleted, bringing the total to 461 observations.

For each simulated data set, we (i) estimate the mimicking portfolio weights d_n in Eq. (17), (ii) calculate ENT using Eqs. (18) and (19), (iii) calculate the portfolio return R_p according to the Eq. (16) but with $b_p = 0$, and finally (iv) estimate the coefficient $\hat{b}_p$ using regression (16).

The result is a collection of 10,800 estimates from which we calculate the p-value.

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